Charting the cQED Design Landscape Using Optimal Control

Michael Goerz[1,2,3], Felix Motzoi[4,5], Birgitta Whaley[5], Christiane P. Koch[1]

[1]Institut für Physik, Universität Kassel, Germany [2]Ginzton Lab, Stanford University [3]Army Research Lab, Adelphi, MD
[4]Theoretische Physik, Universität des Saarlandes, Saarbrücken, Germany [5]Department of Chemistry, UC Berkeley

Quantum Dynamics and Control

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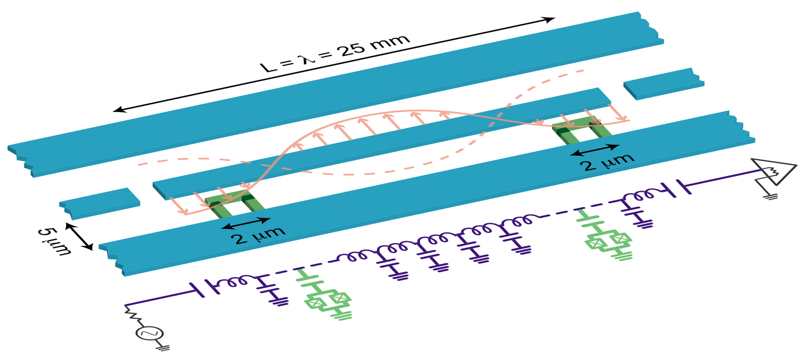
Abstract

Superconducting circuits provide an extremely versatile platform for quantum information processing. Decoherence times have been pushed to tens or hundreds of microseconds, paving the way for large-scale fault tolerant quantum computing. The system parameters may be engineered over a wide range of values. This, however, also provides a considerable challenge in choosing the parameters that most easily allow for the implementation of a universal set of quantum gates. Here, we chart the parameter landscape of the circuit-QED Hamiltonian of two transmon qubits [1] coupled via a shared cavity bus [2]. Using a multi-stage optimal-control procedure, we attempt to find simple control pulses both for a perfectly entangling quantum gate, and a local quantum gate (i.e., a single-qubit gate on each of the qubits), at each point in the parameter space. Gradually decreasing the gate duration allows to estimate the parameter-dependence of the quantum speed limit. We find that the parameter regime that allows for the fastest implementation of gates is outside of the usually considered dispersive regime, prompting the realization of a complete universal set of gates sufficiently fast to beat decoherence.

① Two Transmon Qubits Coupled via Cavity Bus



superconducting qubits inside a transmission line resonator, Fig. from [3]

Parameters:

- $\omega_1 = 6.0$ GHz
- $\omega_2 = 5.0 - 7.5$ GHz (vary)
- $\omega_c = 4.5 - 11.0$ GHz (vary)
- $\alpha_1 = -290$ MHz
- $\alpha_2 = -310$ MHz
- $g = 70$ MHz

$$\hat{\mathbf{H}} = \underbrace{\omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}_{①} + \sum_{q=1,2} \left[\underbrace{\omega_q \hat{\mathbf{b}}_q^\dagger \hat{\mathbf{b}}_q + \frac{\alpha_q}{2} \hat{\mathbf{b}}_q^\dagger \hat{\mathbf{b}}_q^\dagger \hat{\mathbf{b}}_q \hat{\mathbf{b}}_q}_{②} + \underbrace{g(\hat{\mathbf{b}}_q^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_q \hat{\mathbf{a}}^\dagger)}_{③} \right] + \underbrace{\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger}_{④} \qquad (1)$$

with ① the cavity harmonic oscillator, ② qubit anharmonic oscillators, ③ qubit-cavity coupling, and ④ cavity coupling to control field

$$\epsilon(t) = E_0 B(t) \cos(\omega_L t); \qquad B(t) = \text{Blackman shape} \qquad (2)$$

Include spontaneous decay: lifetime of cavity $\tau_c = 3.2\,\mu s$ [4]; lifetime of qubit $\tau_q = 13.3\,\mu s$ [5]

Standard approach: effective model in the *dispersive limit* $|\omega_c - \omega_q| \gg g$. Cavity is only populated virtually, mediates direct effective coupling between qubits and direct driving of qubit excitation.

Here: avoid treating only dispersive regime by numerically solving more general Eq. (1) instead.

② Method

Goal: For each point (ω_2, ω_c): find pulse to maximize entanglement (two-qubit gate) *and* pulse to implement local gate $\in SU(2) \otimes SU(2)$), using multi-stage optimization scheme [6].

1. Random Search

For each point (ω_2, ω_c): random frequencies ω_L, scan amplitude $E_0 \in [10:900]$ MHz.

Look for minimal value of functional $J_{\text{PE}}^{\text{splx}}$ for perfect entangler and $J_{\text{SQ}}^{\text{splx}}$ for arbitrary local gate,

$$J_{\text{PE}}^{\text{splx}} = 1 - C(1 - \varepsilon_{\text{pop}}^{\text{min}}) \qquad (3a)$$
$$J_{\text{SQ}}^{\text{splx}} = 1 - (1 - C)(1 - \varepsilon_{\text{pop}}^{\text{min}}) \qquad (3b)$$

with concurrence C and population loss error $\varepsilon_{\text{pop}}^{\text{min}} = 1 - \min_i \|\hat{\mathbf{U}}\,|i\rangle\,\|$; $\quad |i\rangle \in [00, 01, 10, 11]$.

2. Gradient-free optimization of analytical pulse parameters

For best values of step 1, use Nelder-Mead downhill simplex to minimize Eq. (3) for free pulse parameters E_0, ω_L.

3. Gradient-based optimization (Krotov's method) for fine-tuning

Use Krotov's method [7] to continue optimization of $\epsilon(t)$ for arbitrary perfect entangler [8] and arbitrary local gate $\in SU(2) \otimes SU(2)$, based on Cartan decomposition [9]

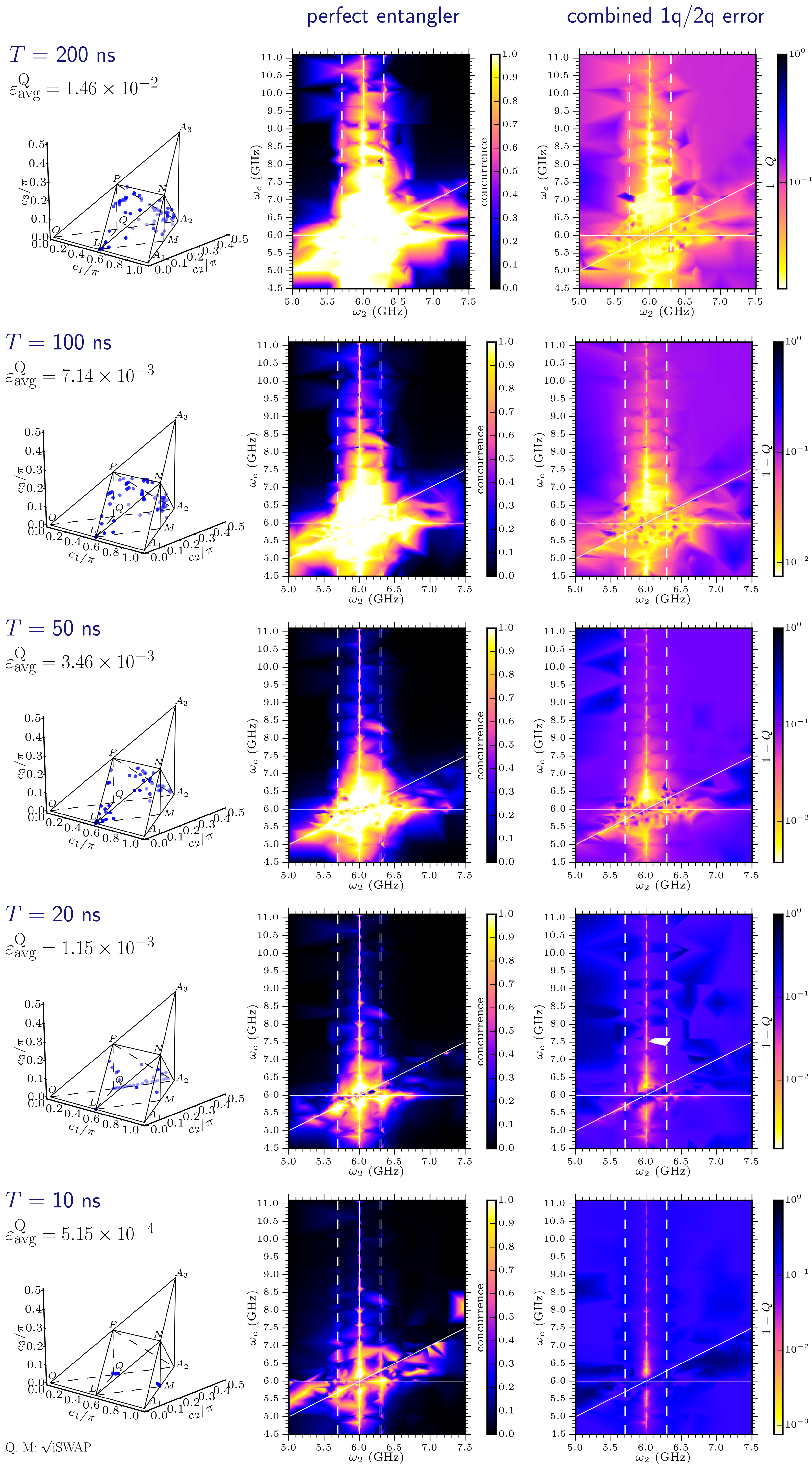
$$\hat{\mathbf{U}} = \hat{\mathbf{k}}_1 \exp\left[\frac{i}{2}\left(c_1 \hat{\boldsymbol{\sigma}}_x \hat{\boldsymbol{\sigma}}_x + c_2 \hat{\boldsymbol{\sigma}}_y \hat{\boldsymbol{\sigma}}_y + c_3 \hat{\boldsymbol{\sigma}}_z \hat{\boldsymbol{\sigma}}_z\right)\right] \hat{\mathbf{k}}_2; \qquad \hat{\mathbf{k}}_{1,2} \in SU(2) \otimes SU(2)$$

Experimentally relevant measure of success for implementing $\hat{\mathbf{O}}$ with $\hat{\mathbf{U}}$ is $F_{\text{avg}} = \int \left\langle \Psi \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{U}} \middle| \Psi \right\rangle \mathrm{d}\Psi$.

"Quality" of a parameter point (ω_2, ω_c) is given by how well an entangling gate *and* a local gate may be implemented.

$$Q(\omega_2, \omega_c) = \frac{1}{2}\Big(F_{\text{avg}}(\hat{\mathbf{O}} = \text{closest PE}) + F_{\text{avg}}(\hat{\mathbf{O}} = \text{closest local gate})\Big); \qquad \varepsilon_{\text{avg}}^{\text{Q}} = 1 - Q$$

③ Charting the Parameter Landscape



Optimization Success (best obtained values)

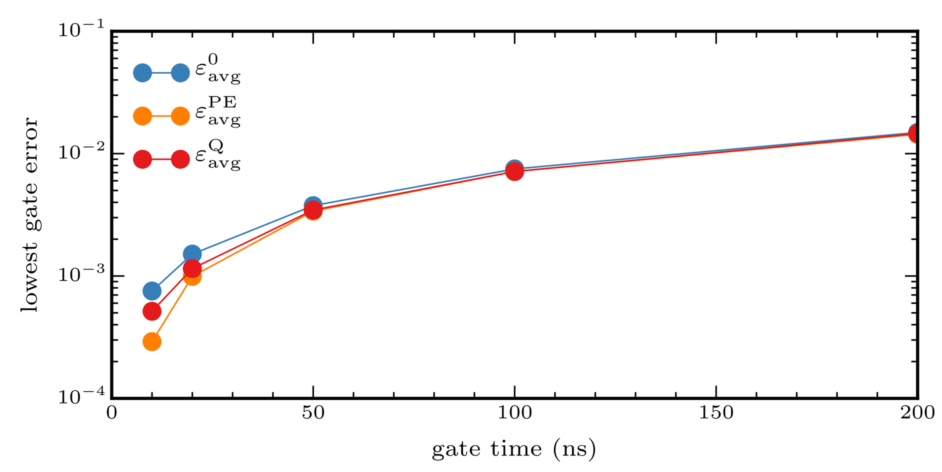
expected error due to dissipation

$$\varepsilon_{\text{avg}}^{0} = 1 - \frac{1}{4}\operatorname{tr}\left[\hat{\mathbf{U}}_0^\dagger \hat{\mathbf{U}}_0\right]$$

with $\hat{\mathbf{U}}_0 = \hat{\mathbf{U}}(g = 0, \epsilon(t) \equiv 0)$

achieved error (PE and combined)

$$\varepsilon_{\text{avg}}^{\text{PE}} = 1 - F_{\text{avg}}(\hat{\mathbf{O}} = \text{closest PE})$$
$$\varepsilon_{\text{avg}}^{\text{Q}} = 1 - Q$$



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④ Conclusions & Outlook

- Found parameters allowing implementation perfect entangler and local gate, for gate durations down to 10 ns, beating decoherence with gate error $< 1 \times 10^{-3}$.
- Obtained gates are limited only by dissipation.
- Fastest gates can be achieved in previously under-explored (non-dispersive) parameter regime with ω_c near ω_1, ω_2.
- Long gate durations allow wide range of two-qubit gates; for short gate durations $\sqrt{\text{iSWAP}}$ is most efficient.
- More complicated pulse shapes than Eq. (2) have been tried, but provide no significant improvement.
- Outlook: implement complete set of universal quantum gates by directly optimizing single-qubit Hadamard and phase gates.
- Analyze characteristics of optimal pulses and dynamics. What gate mechanisms are used?