Efficient Optimal Control for Robust Quantum Gates in Liouville Space

Michael Goerz

Universität Kassel, Germany

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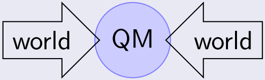
The Problem of Robustness

Problem

Problem of real world quantum engineering: Robustness

- Robustness with respect to decoherence
  $\Rightarrow$ Liouville space
- Robustness with respect to experimental fluctuations and uncertainties



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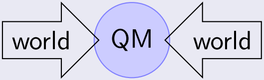

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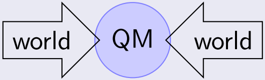

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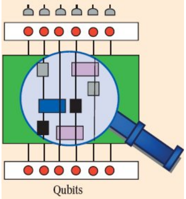
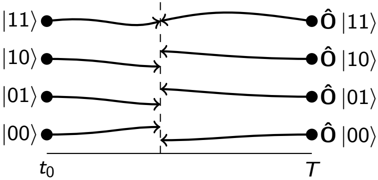
⇒ Optimal Control

- find controls at the quantum speed limit (avoid decoherence)
- ask for robustness *explicitly* in the optimization
  - Optimize in Liouville space... efficiently?
  - Optimize over ensembles of Hamiltonians

Gate Optimization for Open Quantum Systems





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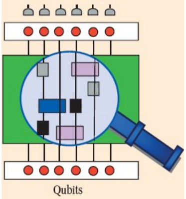

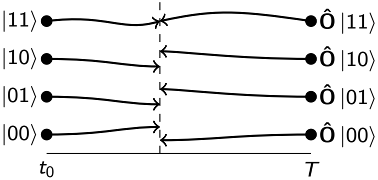

Lift $J_T = 1 - \frac{1}{d} \sum_{i=1}^{d} \mathfrak{Re} \left\langle \Psi_i \middle| \hat{\mathbf{O}}^\dagger \hat{\mathbf{U}}(T, 0, \epsilon) \middle| \Psi_i \right\rangle$ to Liouville space.

Kallush & Kosloff, PRA 73, 032324 (2006),
Schulte-Herbrüggen et al., JPB 44, 154013 (2011),
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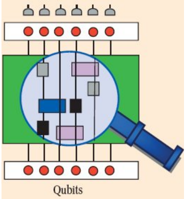

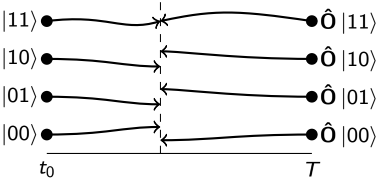

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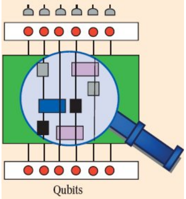
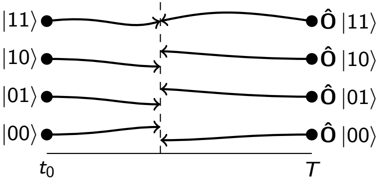
$$\Rightarrow J_t = 1 - \frac{1}{d^2} \sum_{j=1}^{d^2} \mathfrak{Re} \left[\mathrm{tr} \left[\hat{\mathbf{O}} \hat{\rho}_j(0) \hat{\mathbf{O}}^\dagger \, \hat{\rho}_j(T) \right] \right]$$





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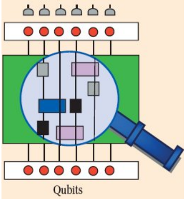
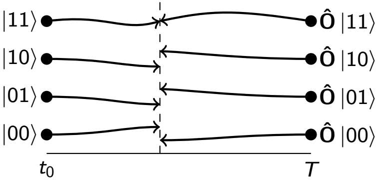
d^2 matrices to propagate! (16 for two-qubit gate)

Gate Optimization for Open Quantum Systems





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Claim

We only need to propagate **three** matrices (independent of d), instead of d^2.

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$$\hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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1. **Do we stay in the logical subspace?**
2. **Are we unitary, and if yes, did we implement the right gate?**

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$$\hat{\rho}_1 = \frac{1}{20}\begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \qquad \hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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E.g. $\hat{\mathbf{O}} = \text{diag}(-1, 1, 1, 1)$;
For $\hat{\mathbf{U}} = \text{diag}(e^{i\phi_{00}}, e^{i\phi_{01}}, e^{i\phi_{10}}, e^{i\phi_{11}})$
using just $\hat{\rho}_1$ will not distinguish $\hat{\mathbf{U}}$ from $\hat{\mathbf{O}}$. ($\hat{\mathbf{U}}\hat{\rho}_1\hat{\mathbf{U}}^\dagger = \hat{\mathbf{O}}\hat{\rho}_1\hat{\mathbf{O}}^\dagger = \hat{\rho}_1$)

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Efficient Gate Optimization in Liouville Space

Optimization States

$$\hat{\rho}_1 = \frac{1}{20}\begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \hat{\rho}_2 = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad \hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

populations | phases | subspace

Functional

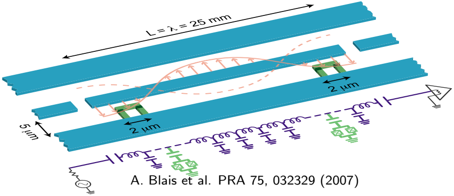
$$J_T = 1 - \sum_{j=1}^{3} \frac{w_j}{\mathrm{tr}[\hat{\rho}_j^2(0)]} \mathfrak{Re}\left[\mathrm{tr}\left[\hat{\mathbf{O}}\hat{\rho}_j\hat{\mathbf{O}}^\dagger\,\mathcal{D}[\hat{\rho}_j]\right]\right]$$

- Allow for different weights ($\sum w_j = 1$)
- $J_T = 0$ if and only if $\forall\ \hat{\rho}_j$: $\mathcal{D}[\hat{\rho}_j] \equiv$ target state
  $\Rightarrow$ implemented unitary gate $\hat{\mathbf{O}}$.

Example 1

Optimizing for Robustness under Dissipation for a Transmon Gate

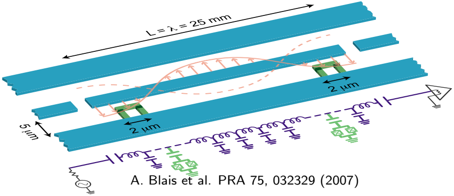
Two Coupled Transmon Qubits



A. Blais et al. PRA 75, 032329 (2007)

Cavity mediates

- driven excitation of qubit
- interaction between left and right qubit



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Effective Hamiltonian

$$\hat{\mathbf{H}}_{\text{eff}} = \sum_{q=1,2} \sum_{i=0}^{N_q-1} (\omega_i^{(q)} + \chi_i^{(q)}) \hat{\mathbf{\Pi}}_i^{(q)} + \sum_{q=1,2} \sum_{i=0}^{N_q-1} g_i^{\text{eff}\,(q)} \epsilon(t) (\hat{\mathbf{C}}_i^{+\,(q)} + \hat{\mathbf{C}}_i^{-\,(q)})$$
$$+ \sum_{ij} J_{ij}^{\text{eff}} (\hat{\mathbf{C}}_i^{-\,(1)} \hat{\mathbf{C}}_j^{+\,(2)} + \hat{\mathbf{C}}_i^{+\,(1)} \hat{\mathbf{C}}_j^{-\,(2)}) \,.$$

with $\omega_i^{(q)} = i\omega_q - \frac{1}{2}(i^2 - i)\alpha_q$, $\quad \hat{\mathbf{\Pi}}_i^{(q)} = \left|i\right\rangle \left\langle i\right|_q$, $\hat{\mathbf{C}}_i^{+\,(q)} = \left|i\right\rangle \left\langle i-1\right|_q$

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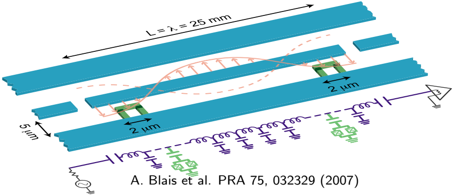

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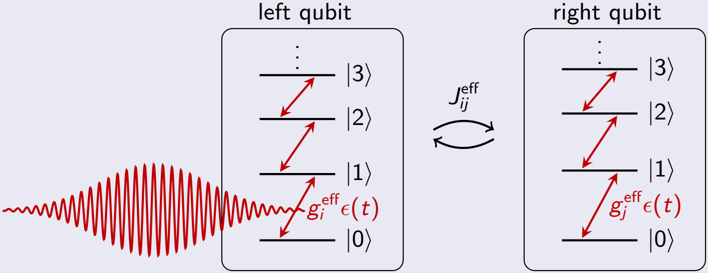

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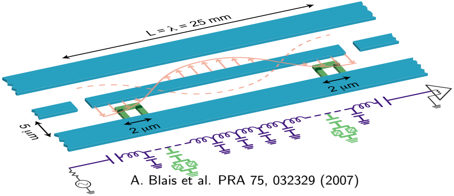

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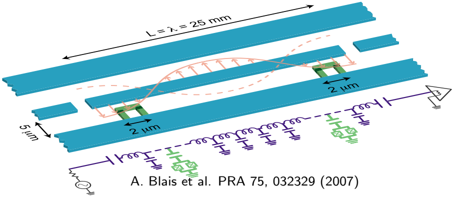
Optimization Target

Many gates possible, e.g. $\sqrt{i\mathrm{SWAP}}$:

$$\hat{\mathbf{O}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Two Coupled Transmon Qubits



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Master Equation

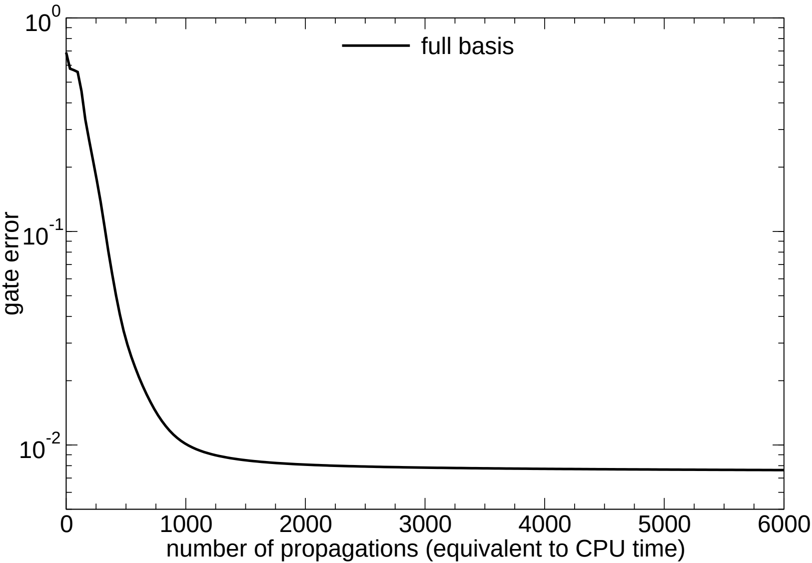
$$\frac{\partial \hat{\rho}}{\partial t} = \frac{i}{\hbar}[\hat{\mathbf{H}}_{\text{eff}}, \hat{\rho}] + \mathcal{L}_D^{(1)}(\hat{\rho}) + \mathcal{L}_D^{(2)}(\hat{\rho})$$

$$\text{with } \mathcal{L}_D^{(q)}(\hat{\boldsymbol{\rho}}) = \gamma_q \sum_{i=1}^{N-1} iD\left[|i-1\rangle\langle i|_q\right]\hat{\boldsymbol{\rho}} + \gamma_{\phi,q}\sum_{i=0}^{N-1}\sqrt{i}D\left[|i\rangle\langle i|_q\right]\hat{\boldsymbol{\rho}}$$

$$\text{with } D\left[\hat{\mathbf{A}}\right]\hat{\rho} = \hat{\mathbf{A}}\hat{\rho}\hat{\mathbf{A}}^\dagger - \tfrac{1}{2}\left(\hat{\mathbf{A}}^\dagger\hat{\mathbf{A}}\hat{\rho} + \hat{\rho}\hat{\mathbf{A}}^\dagger\hat{\mathbf{A}}\right)$$

decay time $T_1 = 38.0$ µs, 32.0 µs; dephasing time $T_2^* = 29.5$ µs, 16.0 µs

Optimization of a Transmon Gate – CPU Time



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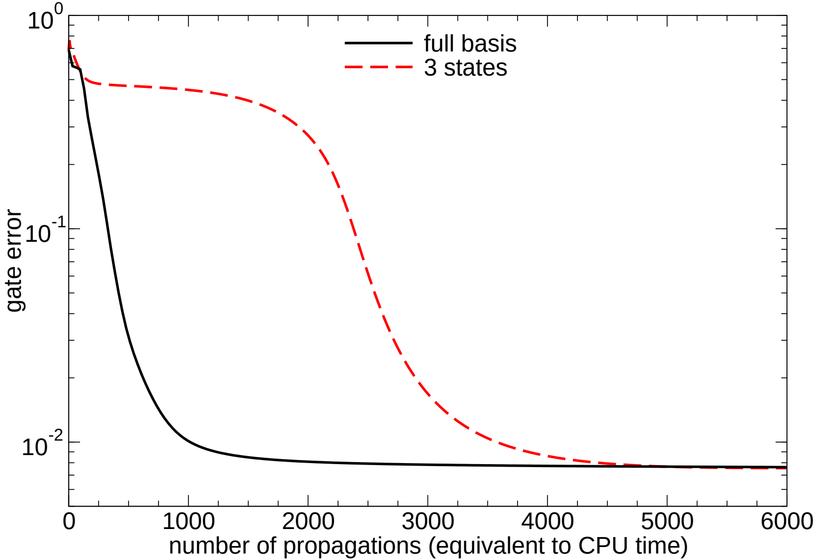

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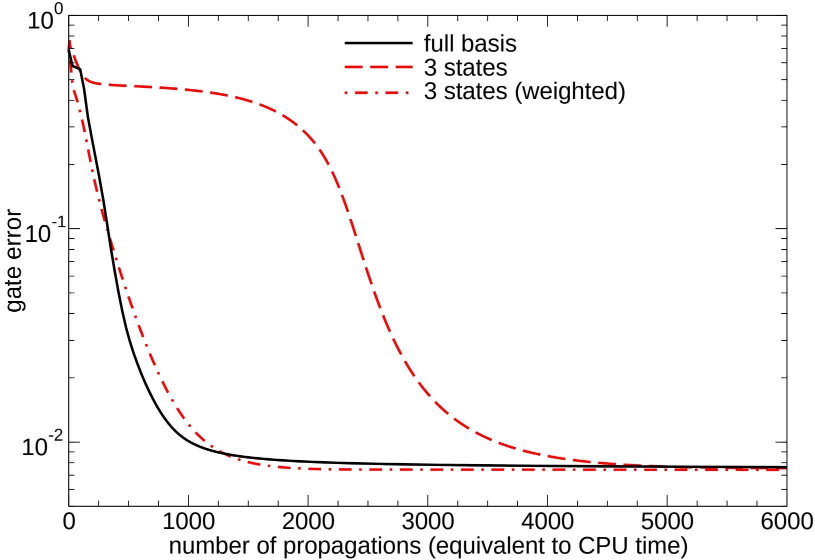

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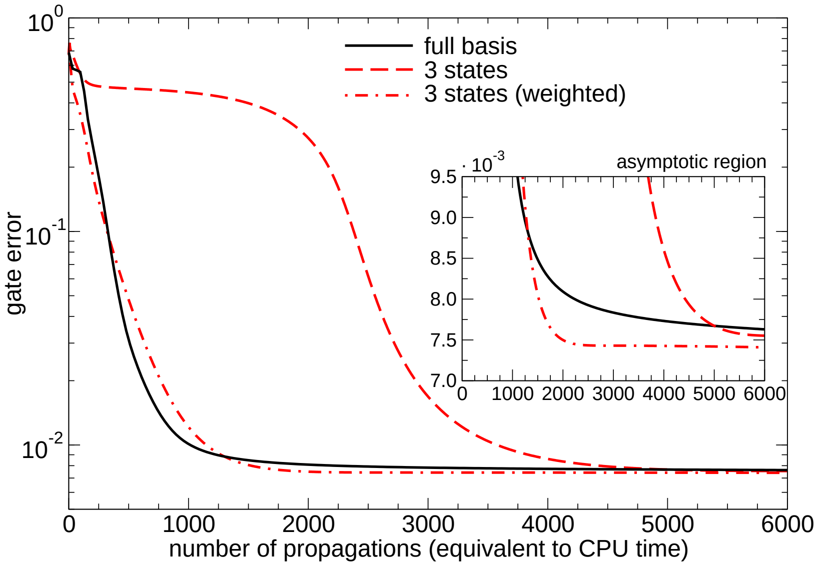

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Using Pure States Only

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$$\Downarrow \qquad\qquad\qquad \searrow$$

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$$\hat{\rho}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; \hat{\rho}_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; \hat{\rho}_5 = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

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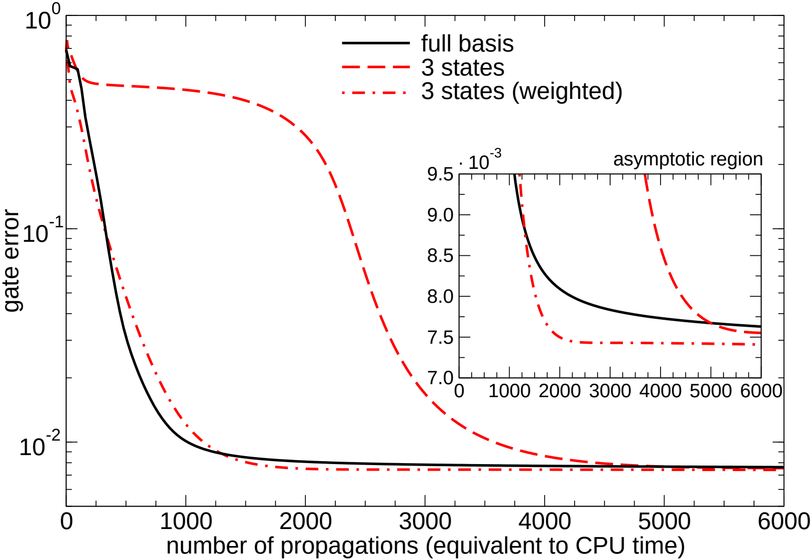

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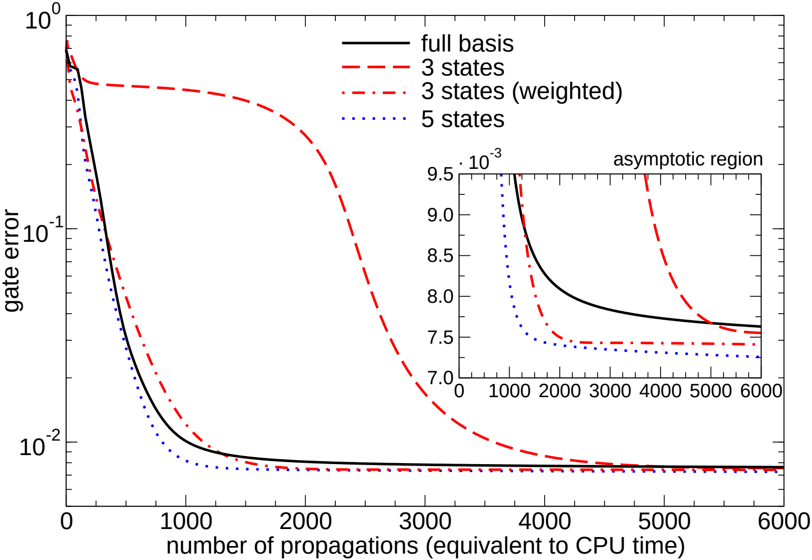

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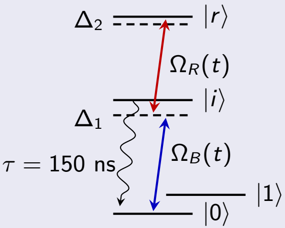
Example 2

Optimization for Robustness under System Fluctuations for a Rydberg Gate

Two Trapped Neutral Atoms

Single-qubit Hamiltonian



In the RWA:

$$\hat{\mathbf{H}}_{1q} = \begin{pmatrix} 0 & 0 & \frac{1}{2}\Omega_R(t) & 0 \\ 0 & E1 & 0 & 0 \\ \frac{1}{2}\Omega_R(t) & 0 & \Delta_1 & \frac{1}{2}\Omega_B(t) \\ 0 & 0 & \frac{1}{2}\Omega_B(t) & 0 \end{pmatrix}$$

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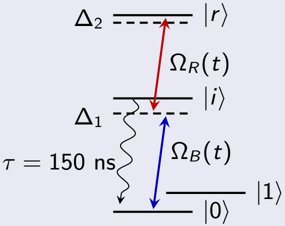

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Two-qubit Hamiltonian

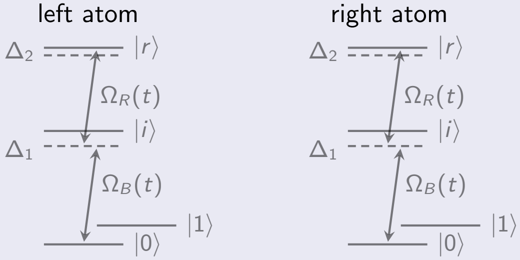
$$\hat{\mathbf{H}}_{2q} = \hat{\mathbf{H}}_{1q} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{H}}_{1q} - U \left|rr\right\rangle \left\langle rr\right|$$

Dipole-dipole interaction when both atoms in Rydberg state.

Only diagonal gates!

Analytical Gate Scheme – Jaksch et al. PRL 85, 2208 (2000)

Two trapped Rydberg atoms



Analytical Gate Scheme – Jaksch et al. PRL 85, 2208 (2000)

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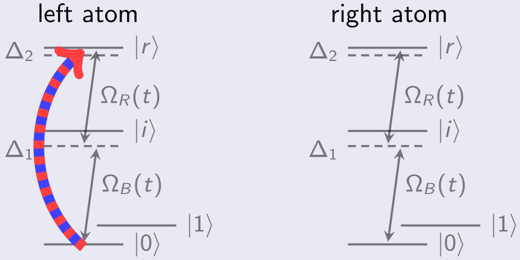

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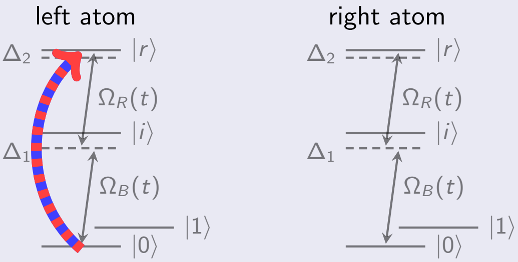
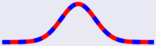
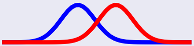


Rabi-pulses in three-level systems

- Option 1:

  two-photon pulse, adiabatic elimination of level $|i\rangle$. 

- Option 2:

  STIRAP. up: ; down: 

$\Rightarrow$ combine both options

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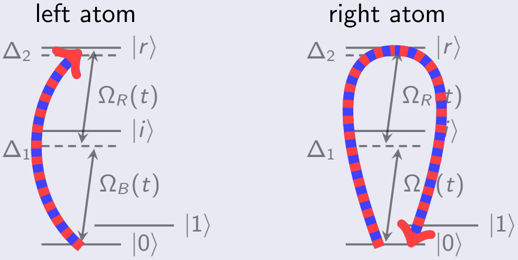

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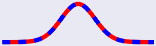
  two-photon pulse, adiabatic elimination of level $|i\rangle$. 
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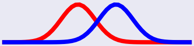
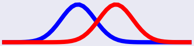
  STIRAP. up: ; down: 

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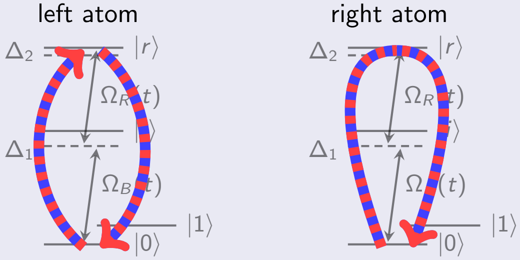

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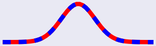
  two-photon pulse, adiabatic elimination of level $|i\rangle$. 

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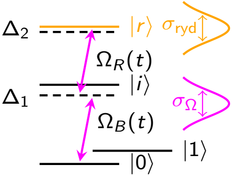
Experimental Fluctuations

Fluctuations:

- Rydberg level (width σ_{ryd}) – external fields
- Pulse amplitude (width σ_{Ω})



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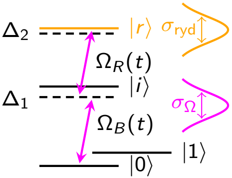

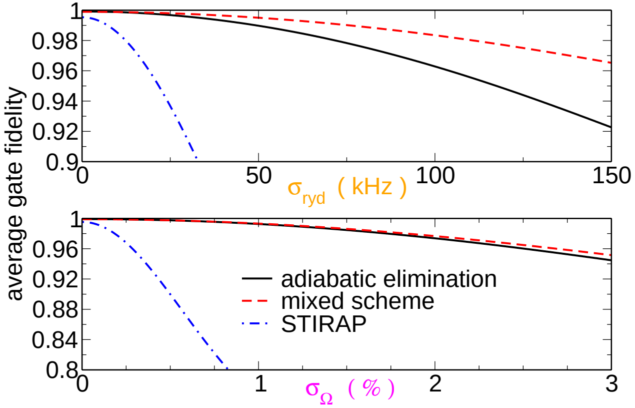

⇒ Goerz, Halperin, Aytac, Koch, Whaley. arXiv:1401.1858.

Optimizing for Robustness

How can OCT improve the robustness?

Optimizing for Robustness

Idea: Ensemble Optimization

Generate set of N Hamiltonians, sampling parameter fluctuations.
Optimize for all Hamiltonians *simultaneously*.

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$$J = 1 - \sum_{n=1}^{N} \sum_{i=1}^{m} \frac{w_{i,n}}{\mathrm{tr}[\hat{\rho}_i^2(0)]} \Re\left\{ \mathrm{tr}\left[\hat{\mathbf{O}} \hat{\rho}_i(0) \hat{\mathbf{O}}^\dagger \hat{\rho}_{i,n}(T) \right] \right\}$$

- N: number of realizations $\hat{\mathbf{H}}_n$: $N = 24$
- m: number of states, for each realizations
- $m \times N$ density matrix propagations!

$$\frac{\partial}{\partial t} \hat{\rho}_{i,n}(t) = -\mathrm{i}\hbar [\hat{\mathbf{H}}_n(t), \hat{\rho}_{i,n}(t)] + \mathcal{L}_D(\hat{\rho}_{i,n}(t))$$

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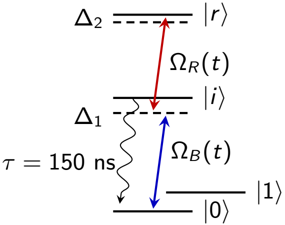
Diagonal Gates



no coupling between $|0\rangle$, $|1\rangle$

$$\hat{\mathbf{U}} = \mathrm{diag}(e^{i\phi_{00}}, e^{i\phi_{01}}, e^{i\phi_{10}}, e^{i\phi_{11}})$$

only diagonal gates are possible

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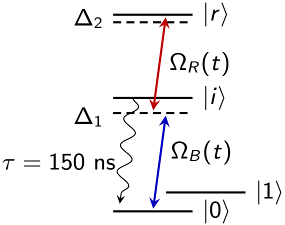

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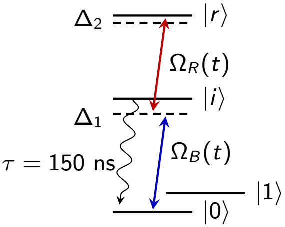

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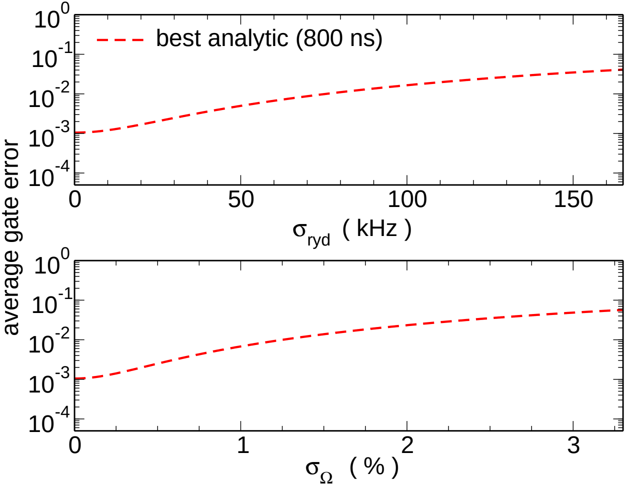
Robustness of OCT Pulses

Again: fluctuation of Rydberg level (σ_{Ryd}) and pulse amplitude (σ_{Ω}).



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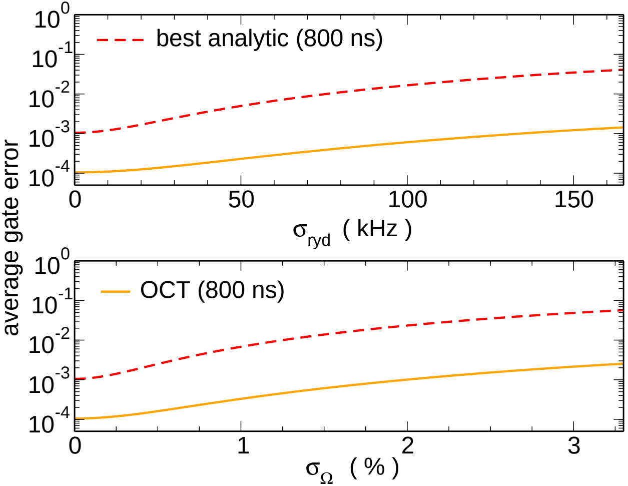

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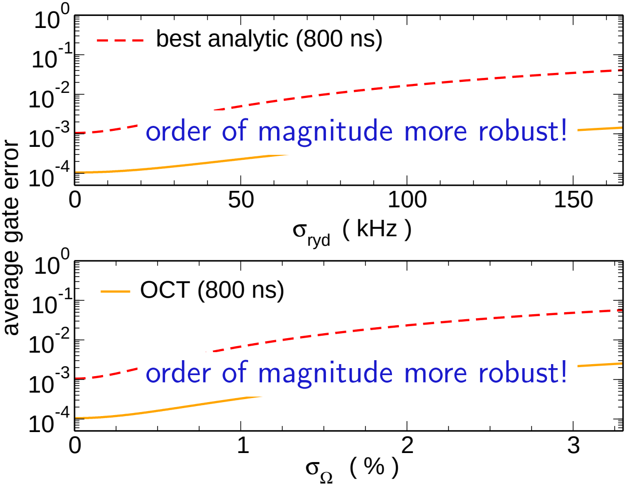

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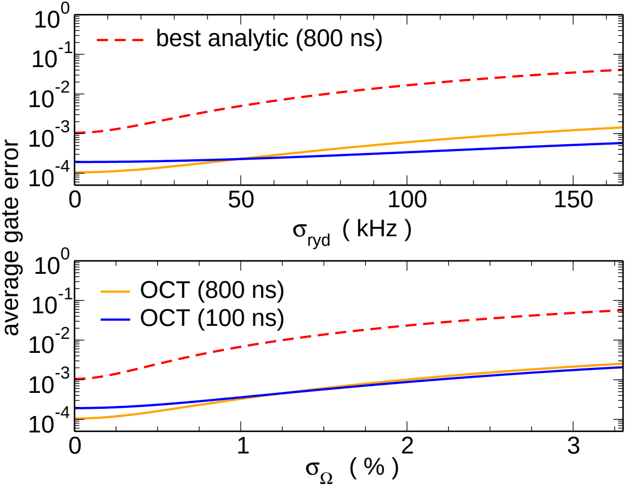

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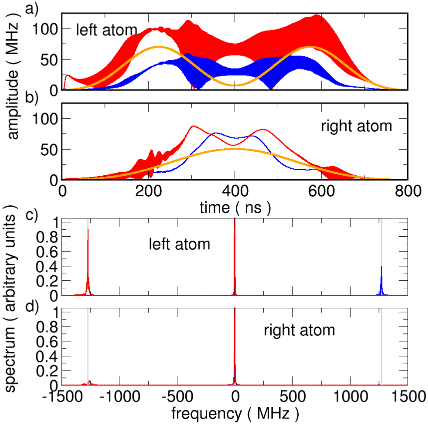
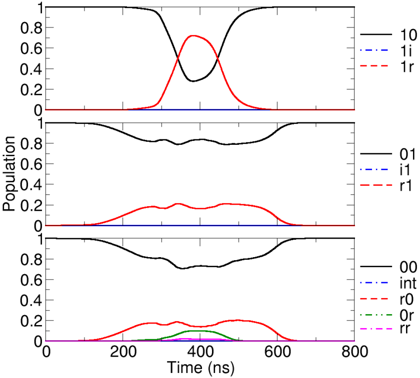
Optimized Pulses & Dynamics

Pulses and Spectra



- STIRAP-like population transfer
- interference between pathways

Population Dynamics



- no population in decaying intermediary states

Conclusion

- Efficiently optimizing for robustness w.r.t. dissipation:
  A set of three density matrices is sufficient
  (independent of dimension of Hilbert space!)
  - one to check dynamical map on subspace
  - one to check the basis
  - one to check the phases

  Further reduction possible for restricted systems!

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Acknowledgments

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- Daniel Reich

Berkeley:

- 
  Birgitta Whaley
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  Eli Halperin
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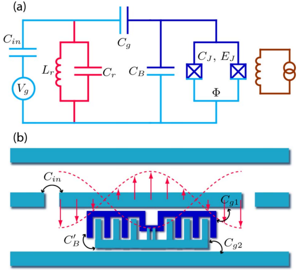
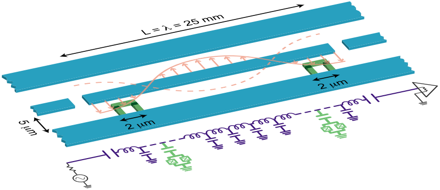
Two Coupled Transmon Qubits



J. Koch et al. PRA 76, 042319 (2007)



A. Blais et al. PRA 75, 032329 (2007)

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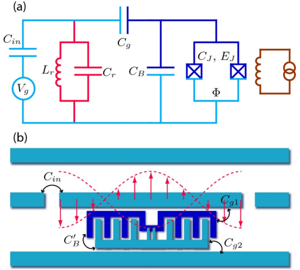

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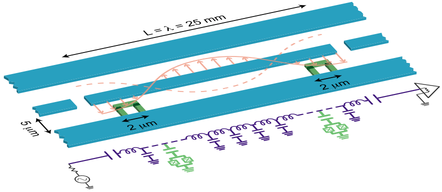

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Full Hamiltonian

$$\hat{\mathbf{H}} = \underbrace{\omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}_{1} + \underbrace{\omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2}_{2} - \underbrace{\frac{1}{2}\left(\alpha_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \alpha_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2\right)}_{3} +$$

$$+ \underbrace{g_1(\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2(\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger)}_{4} + \underbrace{\epsilon^*(t)\hat{\mathbf{a}} + \epsilon(t)\hat{\mathbf{a}}^\dagger}_{5}$$

Effective Hamiltonian

$$
\hat{\mathbf{H}}_{\text{eff}} = \sum_{q=1,2} \sum_{i=0}^{N_q-1} (\omega_i^{(q)} + \chi_i^{(q)}) \hat{\mathbf{\Pi}}_i^{(q)} + \sum_{q=1,2} \sum_{i=0}^{N_q-1} g_i^{\text{eff}\,(q)} \epsilon(t) (\hat{\mathbf{C}}_i^{+\,(q)} + \hat{\mathbf{C}}_i^{-\,(q)})
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with

- $\omega_i^{(q)} = i\omega_q - \frac{1}{2}(i^2 - i)\alpha_q, \qquad g_i^{(q)} = \sqrt{i} g_q$
- $\hat{\mathbf{\Pi}}_i^{(q)} = |i\rangle\langle i|_q$, $\hat{\mathbf{C}}_i^{+\,(q)} = |i\rangle\langle i-1|_q$
- $\chi_i^{(q)} = \frac{(g_i^{(q)})^2}{(\omega_i^{(q)} - \omega_{i-1}^{(q)} - \omega_c)}$
- $g_i^{\text{eff}\,(q)} = \frac{g_i^{(q)}}{(\omega_i^{(q)} - \omega_{i-1}^{(q)} - \omega_c)}$
- $J_{ij}^{\text{eff}} = \frac{1}{2} g_i^{\text{eff}\,(1)} g_j^{(2)} + \frac{1}{2} g_j^{\text{eff}\,(2)} g_i^{(1)}$

IBM Qubit – Poletto et al. PRL 109, 240505 (2012)

qubit frequency ω_1 qubit frequency ω_2 drive frequency ω_d	4.3796 GHz 4.6137 GHz 4.4985 GHz
anharmonicity α_1 anharmonicity α_2	-239.3 MHz -242.8 MHz
effective qubit-qubit coupling J	-2.3 MHz
qubit 1,2 decay time T_1 qubit 1,2 dephasing time T_2^*	38.0 μs, 32.0 μs 29.5 μs, 16.0 μs

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Master Equation

$$\mathcal{L}_D(\hat{\boldsymbol{\rho}}) = \sum_{q=1,2} \left(\gamma_q \sum_{i=1}^{N-1} iD \left[|i-1\rangle\langle i|_q \right] \hat{\boldsymbol{\rho}} + \gamma_{\phi,q} \sum_{i=0}^{N-1} \sqrt{i} D \left[|i\rangle\langle i|_q \right] \hat{\boldsymbol{\rho}} \right) ,$$

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IBM Qubit – Poletto et al. PRL 109, 240505 (2012)

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- Near resonance of α_1 with $\omega_1 - \omega_2$
- single frequency drive centered between two qubits

Effective Hamiltonian

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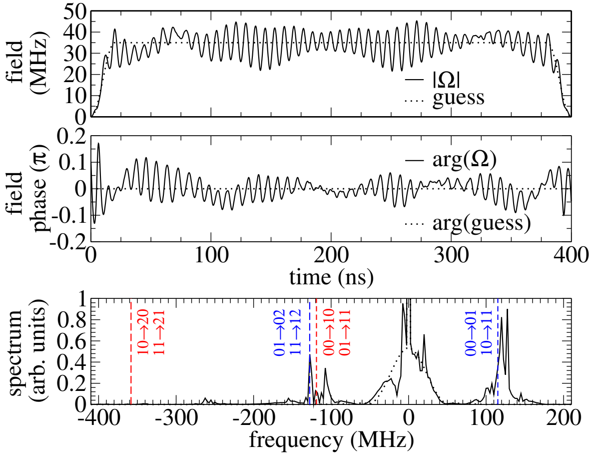
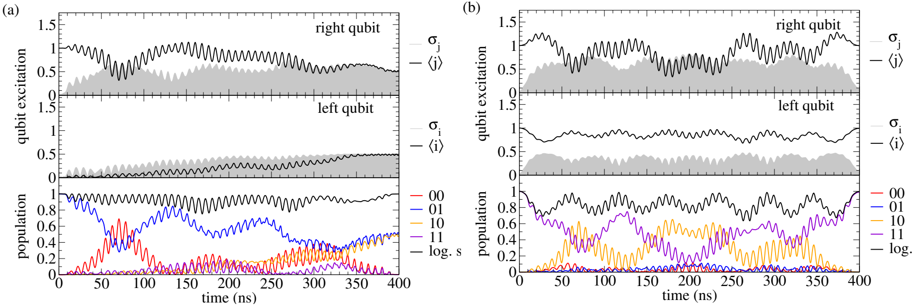
with $D\left[\hat{\mathbf{A}}\right]\hat{\rho} = \hat{\mathbf{A}}\hat{\rho}\hat{\mathbf{A}}^\dagger - \frac{1}{2}\left(\hat{\mathbf{A}}^\dagger\hat{\mathbf{A}}\hat{\rho} + \hat{\rho}\hat{\mathbf{A}}^\dagger\hat{\mathbf{A}}\right)$

Transmon Optimized Pulse



Transmon Population Dynamics

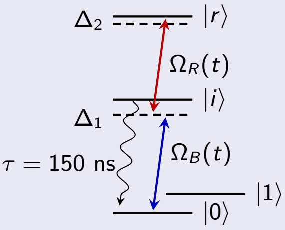


$\Psi(t = 0) = |01\rangle$

$\Psi(t = 0) = |11\rangle$

Analytical Gate Scheme

Single-qubit Hamiltonian



In the RWA:

$$\hat{\mathbf{H}}_{1q} = \begin{pmatrix} 0 & 0 & \frac{1}{2}\Omega_R(t) & 0 \\ 0 & E1 & 0 & 0 \\ \frac{1}{2}\Omega_R(t) & 0 & \Delta_1 & \frac{1}{2}\Omega_B(t) \\ 0 & 0 & \frac{1}{2}\Omega_B(t) & 0 \end{pmatrix}$$

Gate scheme (in blockade regime)

$$\begin{array}{ccccccc} & \pi \text{ (left)} & & 2\pi \text{ (right)} & & \pi \text{ (left)} & \\ |00\rangle & \rightarrow & i\,|r0\rangle & \equiv & i\,|r0\rangle & \rightarrow & -\,|00\rangle \\ |10\rangle & \rightarrow & |10\rangle & \rightarrow & -\,|10\rangle & \rightarrow & -\,|10\rangle \\ |01\rangle & \rightarrow & i\,|r1\rangle & \rightarrow & i\,|r1\rangle & \rightarrow & -\,|01\rangle \\ |11\rangle & \rightarrow & |11\rangle & \rightarrow & |11\rangle & \rightarrow & |11\rangle \end{array}$$