Charting the cQED Design Landscape using Optimal Control Theory

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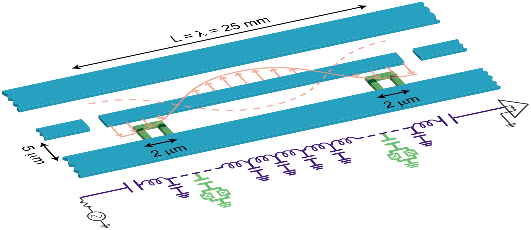
Physical Sciences Seminar
IBM Watson Research Lab
October 23, 2015

two coupled transmon qubits



Blais et al. PRA 75, 032329 (2007)

- each qubit: anharmonic ladder
- coupled to cavity (harmonic)

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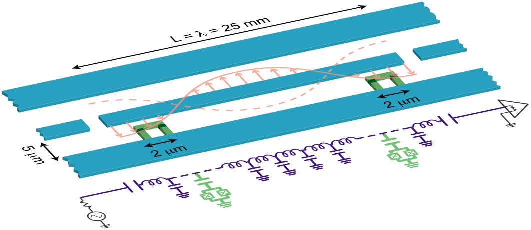

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- tunable qubits
- all-microwave control

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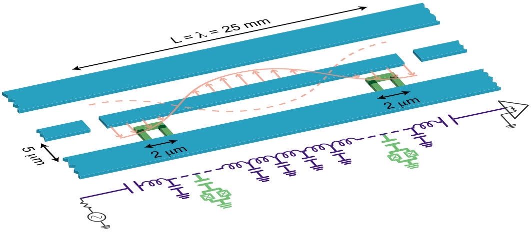

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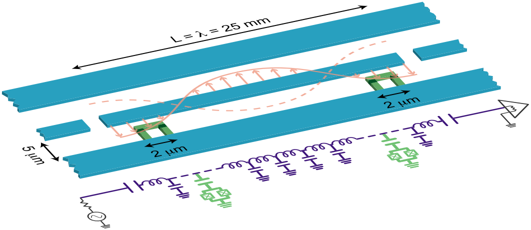

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Full Hamiltonian

$$\hat{\mathbf{H}} = \omega_c \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} + \omega_1 \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 + \omega_2 \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 + \frac{\alpha_1}{2} \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_1 + \frac{\alpha_2}{2} \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2^\dagger \hat{\mathbf{b}}_2 \hat{\mathbf{b}}_2 + g_1 (\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_1 \hat{\mathbf{a}}^\dagger) + g_2 (\hat{\mathbf{b}}_2^\dagger \hat{\mathbf{a}} + \hat{\mathbf{b}}_2 \hat{\mathbf{a}}^\dagger) + \epsilon^*(t) \hat{\mathbf{a}} + \epsilon(t) \hat{\mathbf{a}}^\dagger$$

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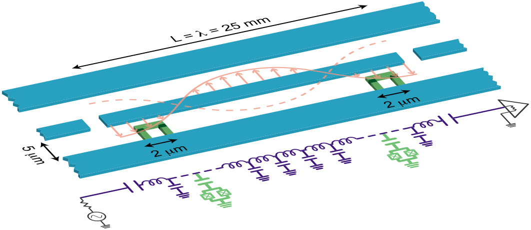

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Logical basis: eigenstates of $\hat{\mathbf{H}}$ ("dressed states")

how should we choose system parameters?

$\omega_1, \omega_2, \omega_c, \alpha_1, \alpha_2, g_1, g_2$ – too many parameters to map out

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physically relevant:

- effective interaction in the dispersive limit
  $\hat{\mathbf{H}} \approx \sum_q \left(\tilde{\omega}_q \hat{\mathbf{n}}_q + \epsilon(t) g^{\text{eff}} (\hat{\mathbf{b}}_q + \hat{\mathbf{b}}_q^\dagger)\right) + g\, g^{\text{eff}} (\hat{\mathbf{b}}_1^\dagger \hat{\mathbf{b}}_2 + \hat{\mathbf{b}}_1 \hat{\mathbf{b}}_2^\dagger);$

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- exploiting resonances
  - $\omega_1 - \omega_2 \approx \alpha_1$ [BR: Poletto et al, PRL 109, 240505]
  - $\omega_1 - \omega_2 \approx 2\alpha_1$ [MAP: Chow et al, NJP 15, 115012]

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$\Rightarrow \omega_1 = 6$ GHz, $\alpha_1 = -290$ MHz, $\alpha_2 = -310$ MHz, $g = 70$ MHz
vary: $4.5 < \omega_c < 11.0$ GHz; $5.0 < \omega_2 < 7.5$ GHz

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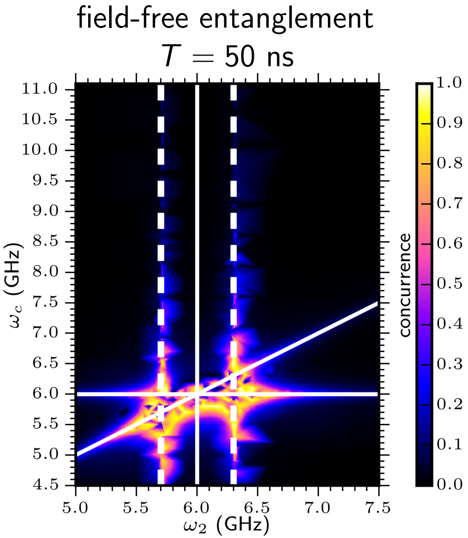
Dissipation: $\tau_c = 3.2\ \mu$s, $\tau_q = 13.3\ \mu$s

the parameter landscape



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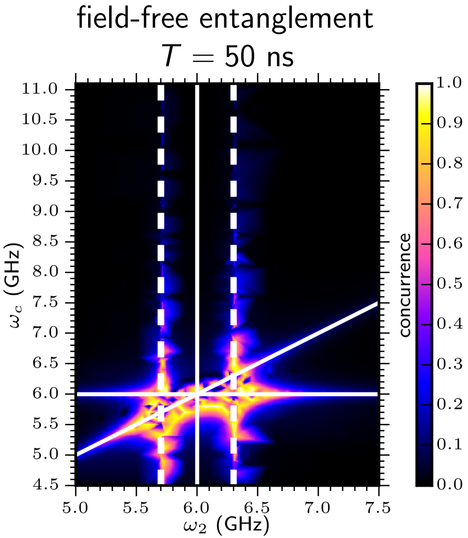

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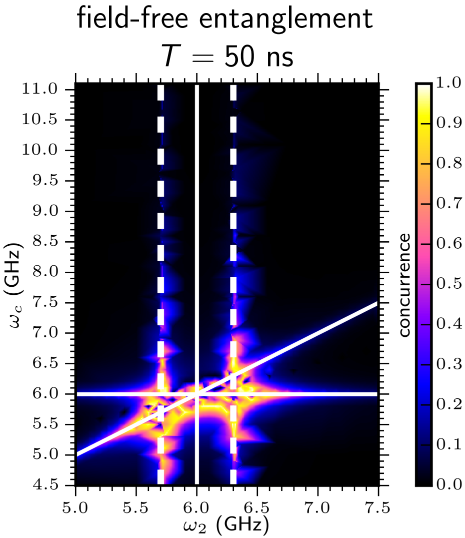

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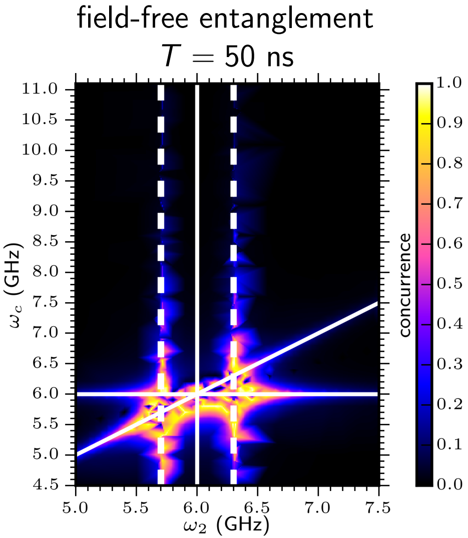


- entangling gates: choose $\epsilon(t)$ so that $C \to 1$
- local gates: choose $\epsilon(t)$ so that $C \to 0$
- what is the speed limit?

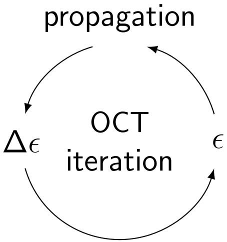
What is numerical optimal control all about?

numerical optimal control

- solve equation of motion numerically

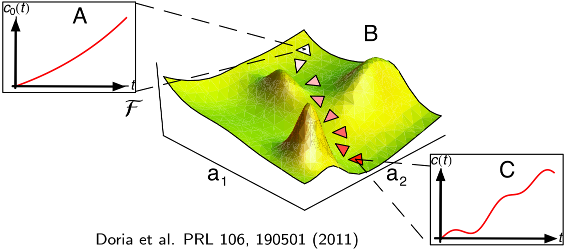
$$\hat{\mathbf{U}}(dt) = e^{-i\hat{\mathbf{H}}dt} \quad \Rightarrow \text{expand in Chebychev Polynomials}$$
$$\mathcal{E}(dt) = e^{-i\mathcal{L}dt} \quad \Rightarrow \text{expand in Newton Polynomials}$$

- iteratively improve control $\epsilon(t)$



gradient-free optimization

Take into account *only* evaluation of figure of merit.



Doria et al. PRL 106, 190501 (2011)

e.g. Nelder-Mead (simplex), genetic algorithms…

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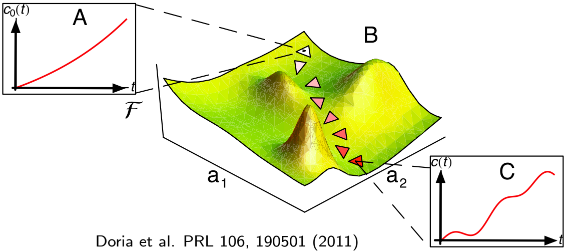


Doria et al. PRL 106, 190501 (2011)

e.g. Nelder-Mead (simplex), genetic algorithms...

advantages:

- any figure of merit
- easy to use in experiment

disadvantages:

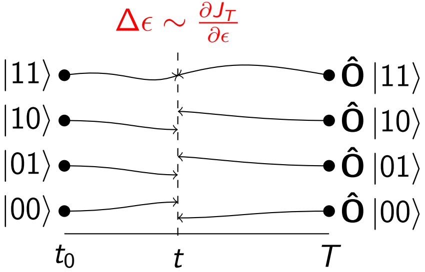
- only for low-dimensional search space

gradient-based optimization

Take into account *derivative* of figure of merit

- gradient descent/LBFGS: concurrent scheme, needs $\frac{\partial J_T}{\partial \epsilon}$
- Krotov's method: sequential scheme, needs $\frac{\partial J_T}{\partial \langle \Psi |}$

Reich et al. JCP 136, 104103 (2012)



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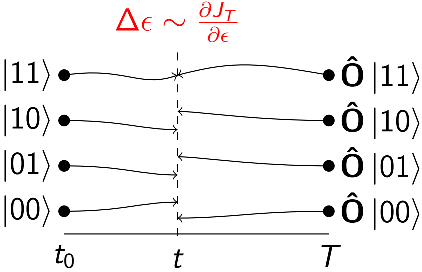

advantages:

- fast convergence
- high-dimensional search space

disadvantages:

- fig. of merit not arbitrary
- numerically expensive

hybrid optimization schemes

1. Start with analytical formula, optimize free parameter with simplex
2. Use simplex-optimized control as starting point for gradient-based method

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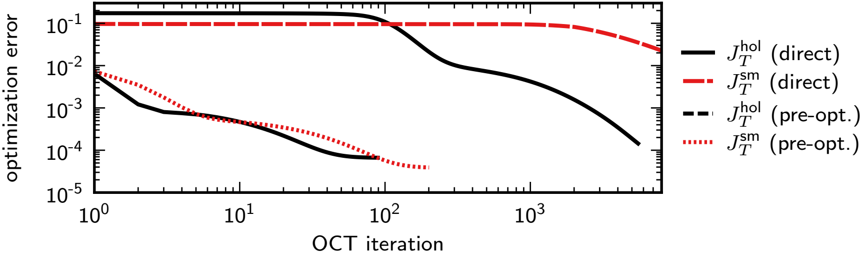

Goerz et al. EPJ Quantum Tech. 2, 21 (2015)

Optimizing quantum gates

functionals for quantum gates

Hilbert space

$$J_T = 1 - \frac{1}{16} \left| \sum_{i=1}^{4} \left\langle i \,\middle|\, \hat{\mathbf{O}}^\dagger \hat{\mathbf{U}} \,\middle|\, i \right\rangle \right|^2 ; \quad \left|i\right\rangle \in \{\left|00\right\rangle, \left|01\right\rangle, \left|10\right\rangle, \left|11\right\rangle\}$$

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Liouville space ⇒ Goerz et al. NJP 16, 055012 (2014)

$$J_T = 1 - \sum_{i=1}^{3} \frac{w_i}{\mathrm{tr}[\hat{\boldsymbol{\rho}}_i^2]} \mathfrak{Re}\left\{\mathrm{tr}\left[\hat{\boldsymbol{\rho}}_i^{\mathrm{tgt}} \hat{\boldsymbol{\rho}}_i(T)\right]\right\}$$

$$\hat{\rho}_1 = \frac{1}{20}\begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}, \quad \hat{\rho}_2 = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad \hat{\rho}_3 = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

populations phases subspace

functionals for quantum gates

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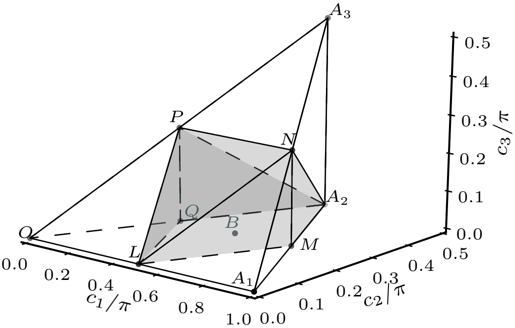
populations   phases   subspace

⇒ more advanced functionals

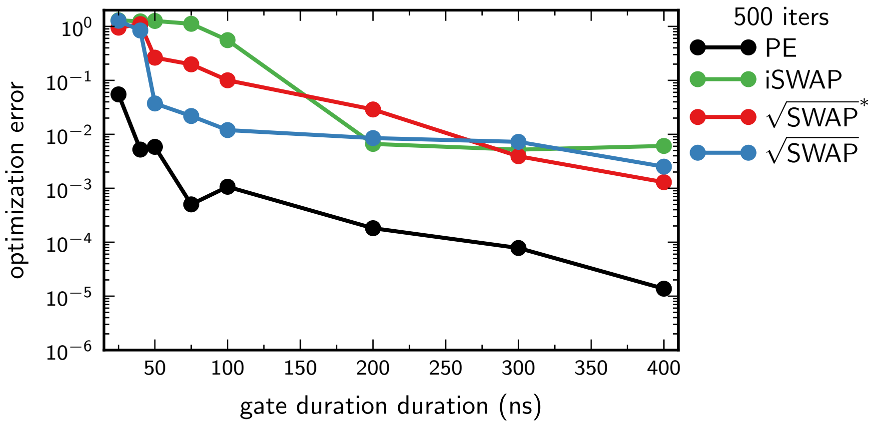
quantum gates in the Weyl chamber



Cartan Decomposition

$$\hat{\mathbf{U}} = \hat{\mathbf{k}}_1 \exp\left[\frac{\mathrm{i}}{2}\left(c_1 \hat{\boldsymbol{\sigma}}_x \hat{\boldsymbol{\sigma}}_x + c_2 \hat{\boldsymbol{\sigma}}_y \hat{\boldsymbol{\sigma}}_y + c_3 \hat{\boldsymbol{\sigma}}_z \hat{\boldsymbol{\sigma}}_z\right)\right] \hat{\mathbf{k}}_2$$

optimizing for an arbitrary perfect entangler



$\Rightarrow$ Watts et al. PRA 91, 062306 (2015)
Goerz et al. PRA 91, 062307 (2015)

Charting the transmon parameter landscape

procedure

At each point (ω_c, ω_1), for control $\epsilon(t) = E_0 B(t) \cos(\omega_L t)$:

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1. random search

$$J_{\text{PE}}^{\text{splx}} = 1 - C(1 - \varepsilon_{\text{pop}}^{\text{min}}), \quad J_{\text{SQ}}^{\text{splx}} = 1 - (1 - C)(1 - \varepsilon_{\text{pop}}^{\text{min}})$$

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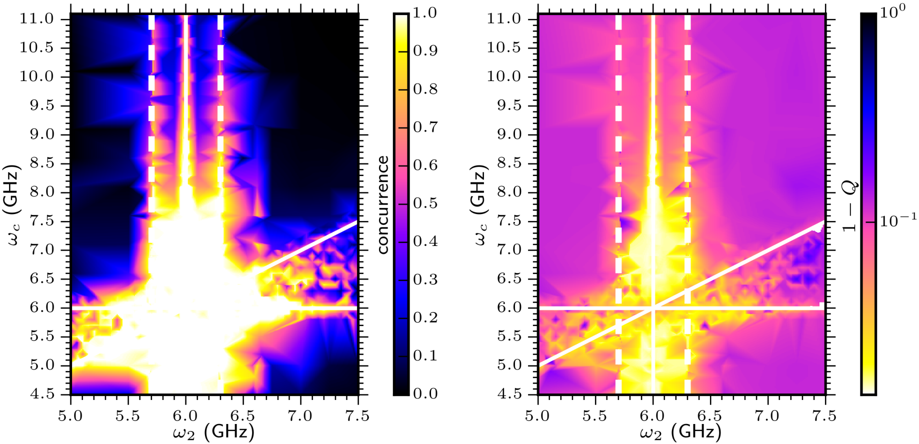
Evaluate success via $F_{\text{avg}} = \int \left\langle \Psi \middle| \hat{\mathbf{O}}^{\dagger} \hat{\mathbf{U}} \middle| \Psi \right\rangle \mathrm{d}\Psi$.

"Quality": $Q = \frac{1}{2}\left(F_{\text{avg}}(\hat{\mathbf{O}} = \text{PE}) + F_{\text{avg}}(\hat{\mathbf{O}} = \text{SQ})\right)$

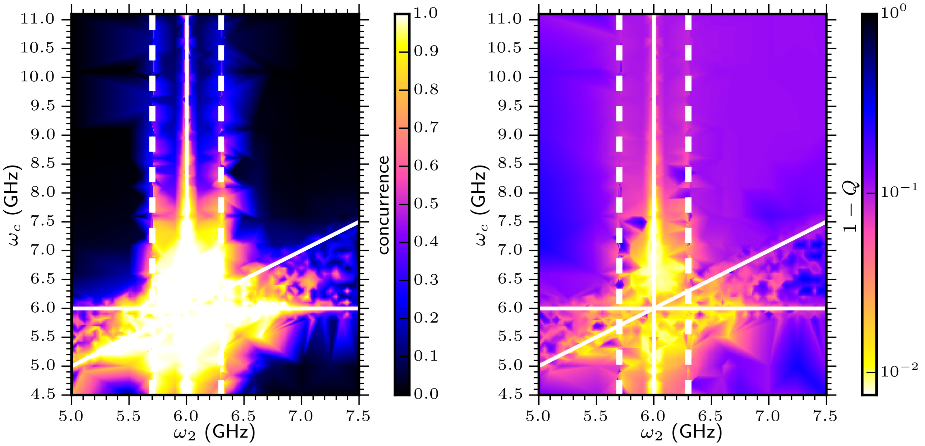
how well can we do local/entangling gates?

$T = 200$ ns



how well can we do local/entangling gates?

$T = 100$ ns



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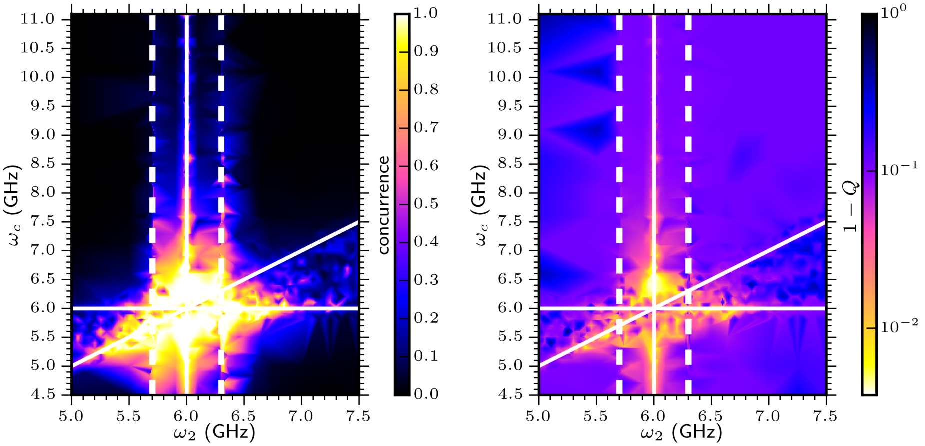
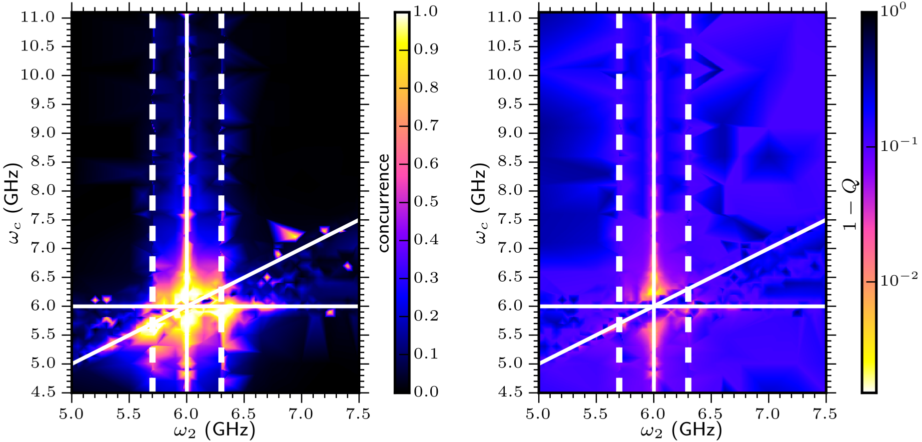
$T = 50$ ns



how well can we do local/entangling gates?

$T = 20$ ns



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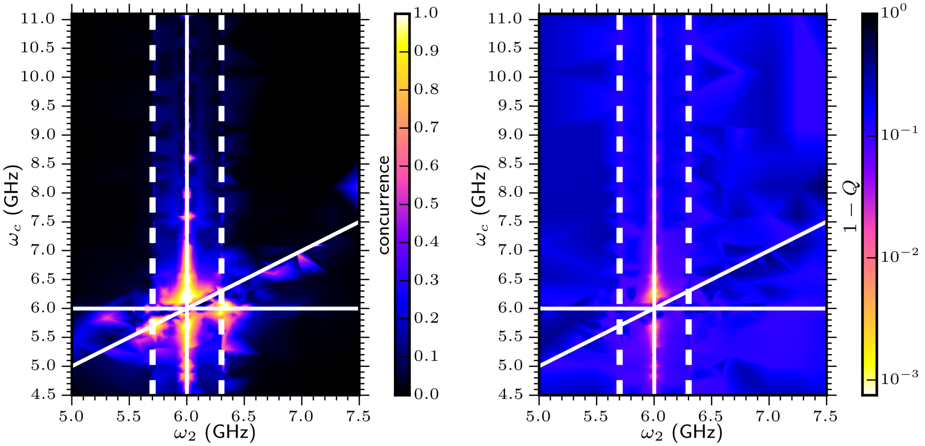
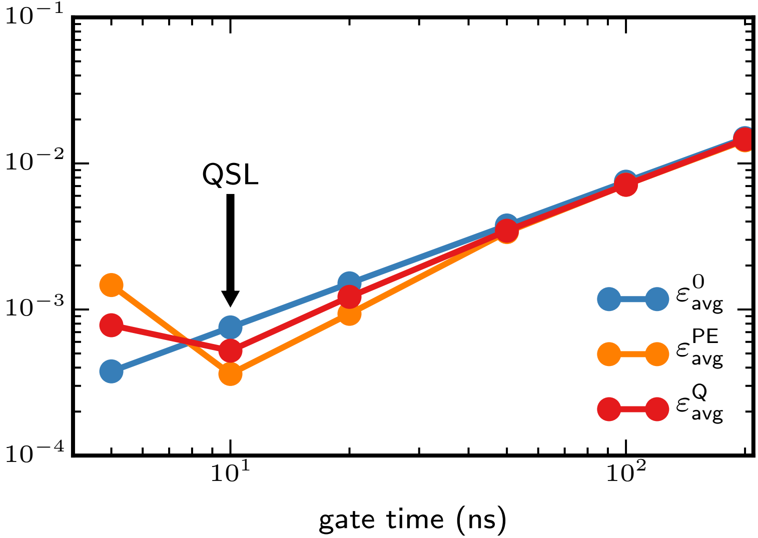
$T = 10$ ns

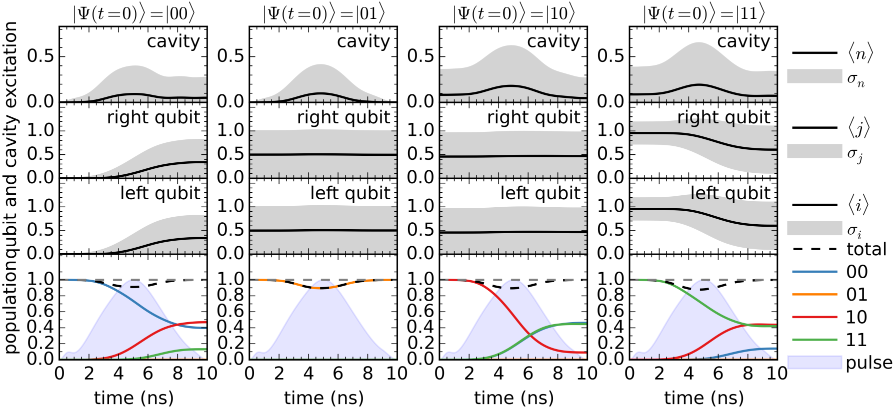


quantum speed limit for all-microwave control



population dynamics for a perfect entangler

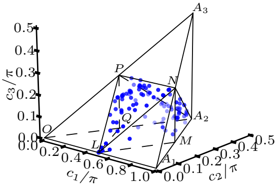
$$\omega_1 = \omega_2 = 6.0\ \mathrm{GHz},\ \omega_c = 6.3\ \mathrm{GHz}$$



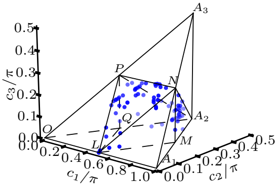
$$E_0 = 215.7\ \mathrm{MHz},\ \omega_L = 5.964\ \mathrm{GHz} \quad \Rightarrow \epsilon_{\mathrm{avg}} = 1.3 \times 10^{-3}$$

obtained gates

$T = 200$ ns



$T = 100$ ns



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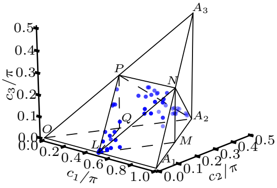
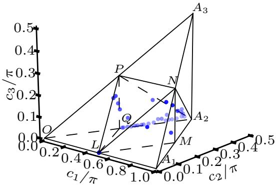


$T = 20$ ns



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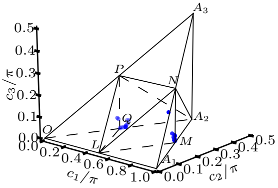
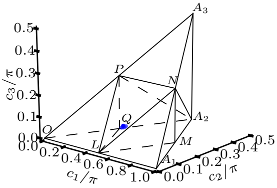


$T = 5$ ns



summary & conclusion

Conclusions

- Optimal control can be tool for systematic parameter exploration
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Thank you!