Efficient Numerical Optimization via Quantum Trajectories

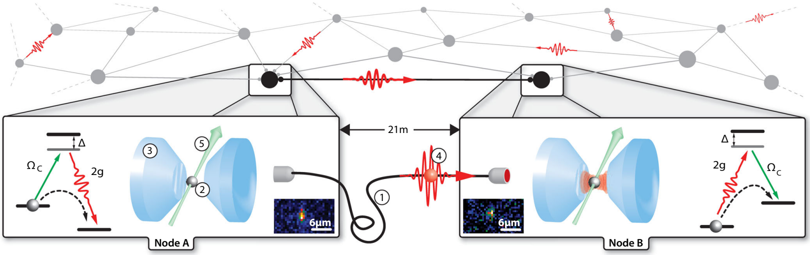
Michael Goerz

Stanford University / Army Research Lab

Gordon Research Conference
Quantum Control of Light & Matter

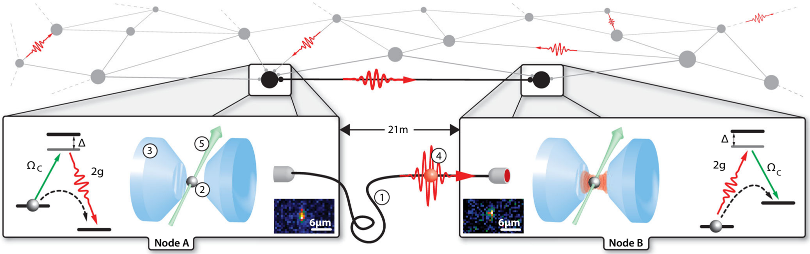
Mount Holyoke College
August 6, 2017

quantum networks



from: Reiserer, Rempe. RMP 87, 1379 (2015)

quantum networks



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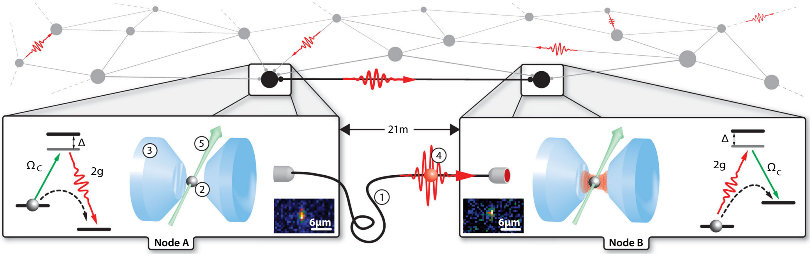
applications of networks

quantum networks



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- secure qu. communication

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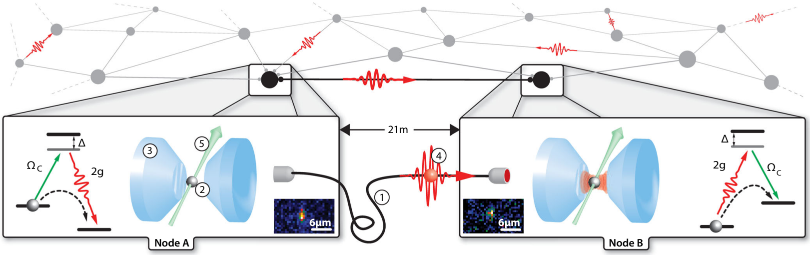

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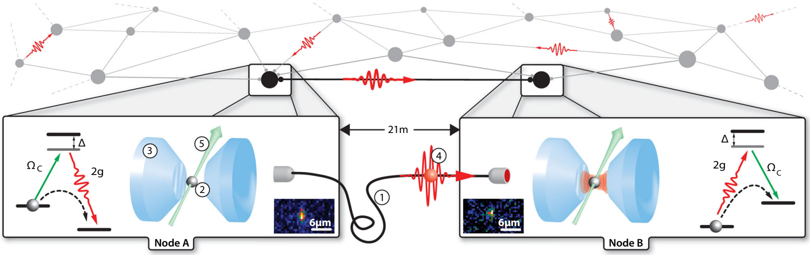

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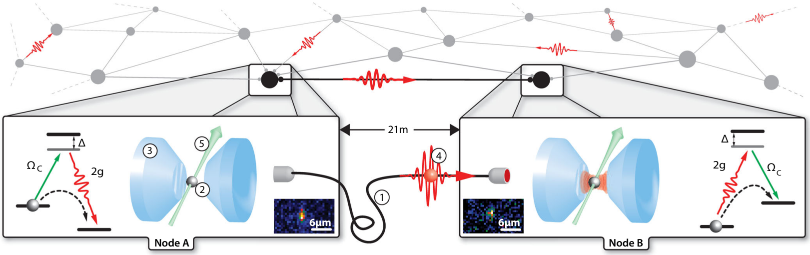

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network advantages:
scaling, robustness, security

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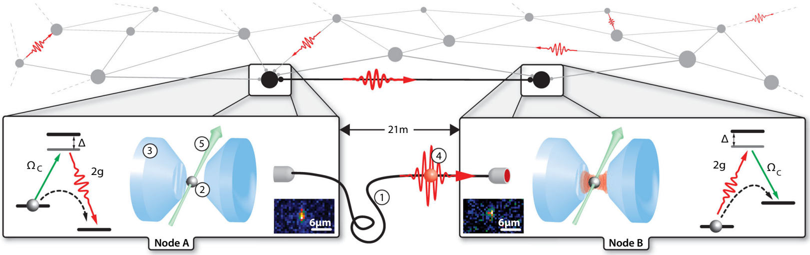

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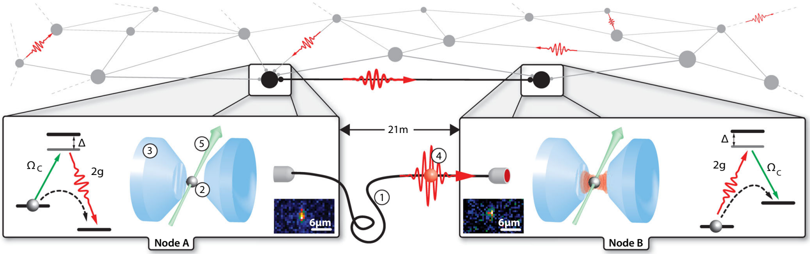

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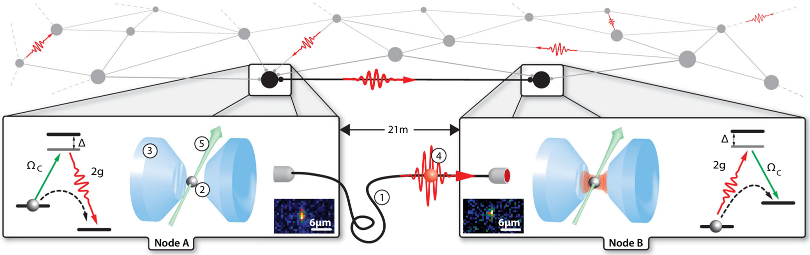

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- node-link interfaces
- entanglement creation & distribution

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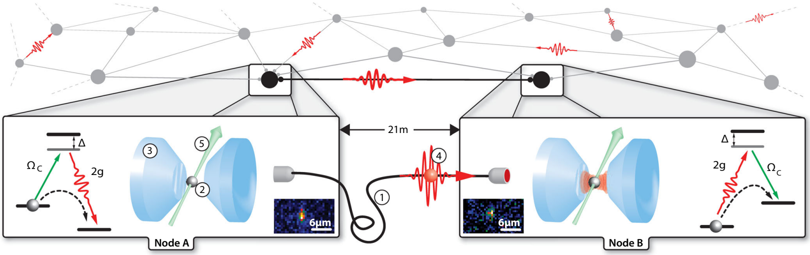

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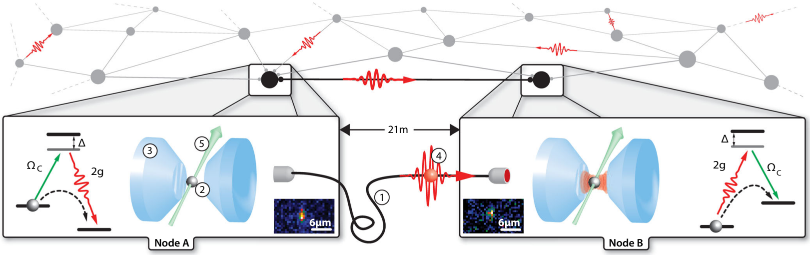

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- node-link interfaces
- entanglement creation & distribution
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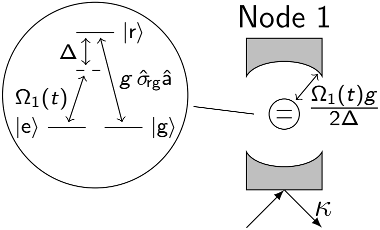
- local processing

network model

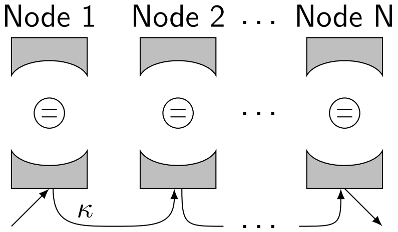


single node:

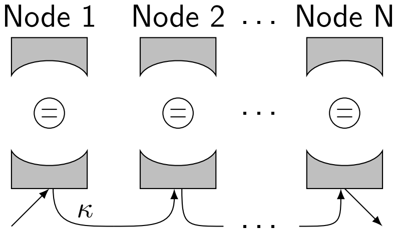
$$\hat{\mathbf{H}}^{(1)} = \hat{\mathbf{H}}_0^{(1)} + u_1(t)\left(\hat{\boldsymbol{\sigma}}_{eg} \otimes \hat{\mathbf{a}}_1^\dagger + \text{c.c.}\right)$$

$$\hat{\mathbf{L}}^{(1)} = \sqrt{2\kappa}\,\hat{\mathbf{a}}_1$$

cf. Cirac et al, PRL 78, 3221 (1997)


Node 1
Node 2
. . .
Node N
. . .
. . .
κ

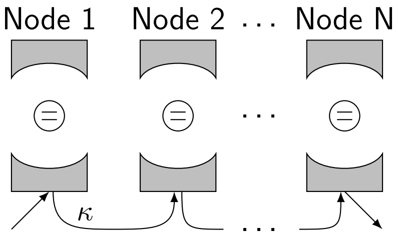
network model



SLH formalism (Gough, James):

$$\{(\hat{\mathbf{H}}^{(i)}, \{\hat{\mathbf{L}}^{(i)}\})\} \text{ of nodes} \rightarrow (\hat{\mathbf{H}}, \{\hat{\mathbf{L}}\}) \text{ of network}$$

network model



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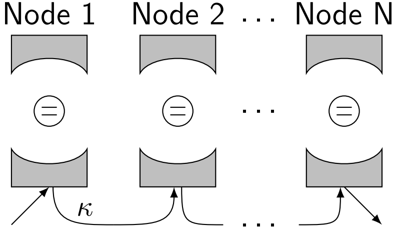
QNET: computer (quantum) algebra software

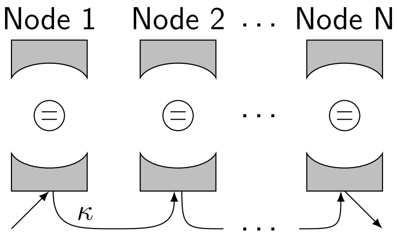
`https://github.com/mabuchilab/QNET`

network model



$$\hat{\mathbf{H}} = \sum_{i=1}^{N} \hat{\mathbf{H}}^{(i)} + \sum_{i \neq j} \mathrm{i}\kappa \hat{\mathbf{a}}_i^\dagger \hat{\mathbf{a}}_j + \text{c.c}\,; \qquad \hat{\mathbf{L}} = \sqrt{2\kappa} \sum_{i=1}^{N} \hat{\mathbf{a}}_i$$

network model



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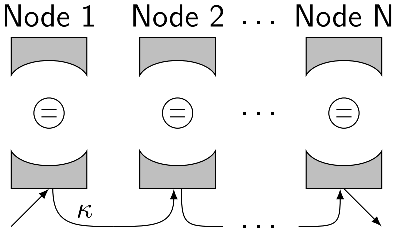

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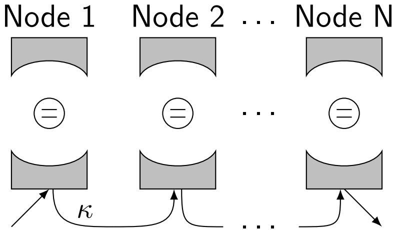
- Hilbert space dimension exponential with N

network model



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- node-node interaction and decay both $\propto \kappa$

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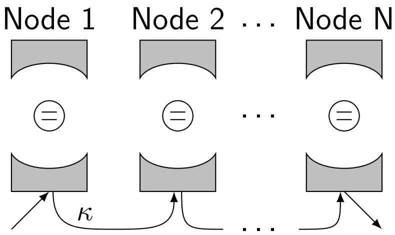

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- Hilbert space dimension exponential with N
- node-node interaction and decay both $\propto \kappa$
  $\Rightarrow$ dark state $\left\langle \Psi \middle| \hat{\mathbf{L}}^\dagger \hat{\mathbf{L}} \middle| \Psi \right\rangle = 0$

quantum jump method (MCWF)

see Dum et al. PRA 4879 (1992); Mølmer et al. JOSAB 10, 524 (1993)

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2. random number $r \in [0, 1)$;
   propagate until $|\Psi_k(t)|^2 = r$.

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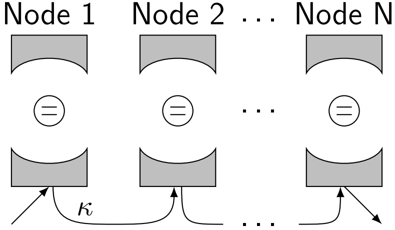
trajectory averaging:

$$\hat{\rho} = \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{k=1}^{M} \left|\Psi_k\right\rangle \left\langle \Psi_k\right|$$

generating entangled Dicke state



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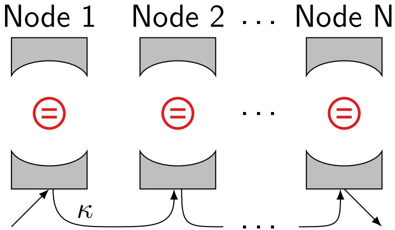

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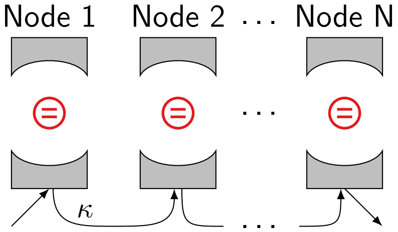

Dicke state: distribute a fixed excitation number over all nodes

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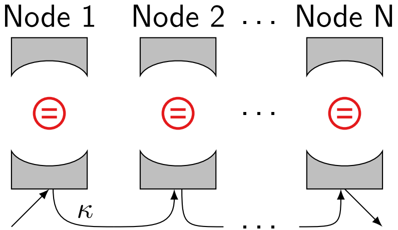
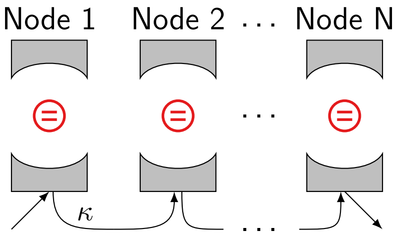


Dicke state: distribute a fixed excitation number over all nodes

for single excitation:

$$|10\dots0\rangle \rightarrow \frac{1}{\sqrt{N}}\left(|10\dots0\rangle + |01\dots0\rangle + \dots + |00\dots1\rangle\right)$$

generating entangled Dicke state



optimal control problem

find control fields $u_1(t) \dots u_N(t)$
that drive $|\Psi(0)\rangle \rightarrow |\Psi(T)\rangle = |\Psi\rangle_{\text{tgt}}$

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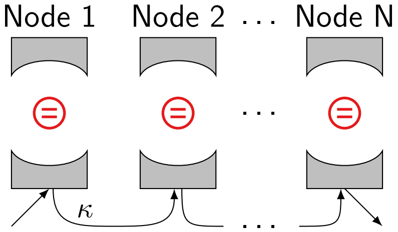

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minimize $\quad J_T = 1 - \langle\langle \hat{\rho}(T) | \hat{\mathbf{P}}_{\text{tgt}} \rangle\rangle$

gradient optimization

$$\text{minimize} \qquad J_T = 1 - \left\langle\!\left\langle \hat{\rho}(T) \middle| \hat{\mathbf{P}}_{\text{tgt}} \right\rangle\!\right\rangle$$

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GRAPE/LBFGS Khaneja et al, JMR 172, 296 (2005)

on time grid: $u_{ij} = u_i(t_j)$; $\qquad \mathcal{E}_j = \mathcal{E}(t_j, t_{j-1})$

$$\Delta u_{ij} \propto \frac{\partial J_T}{\partial u_{ij}} = -\left\langle\!\!\left\langle \hat{\mathbf{P}}^{(0)}(t_j) \middle| \frac{\partial \mathcal{E}_j}{\partial u_{ij}} \middle| \hat{\rho}^{(0)}(t_{j-1}) \right\rangle\!\!\right\rangle,$$

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Krotov's method

Reich et al, JCP 136, 104103 (2012)

aux. functional $J = J_T + \sum_i \frac{\lambda_i}{S_i(t)} \int_0^T [u_i^{(1)}(t) - u_i^{(0)}(t)]^2 \, \mathrm{d}t$

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on time grid: $u_{ij} = u_i(t_j)$; $\qquad \mathcal{E}_j = \mathcal{E}(t_j, t_{j-1})$

$$\Delta u_{ij} \propto \frac{\partial J_T}{\partial u_{ij}} = -\left\langle\!\!\left\langle \hat{\mathbf{P}}^{(0)}(t_j) \middle| \frac{\partial \mathcal{E}_j}{\partial u_{ij}} \middle| \hat{\rho}^{(0)}(t_{j-1}) \right\rangle\!\!\right\rangle,$$

Krotov's method

Reich et al, JCP 136, 104103 (2012)

aux. functional $J = J_T + \sum_i \frac{\lambda_i}{S_i(t)} \int_0^T [u_i^{(1)}(t) - u_i^{(0)}(t)]^2 \, \mathrm{d}t$

$$\Delta u_i(t) = \frac{S_i(t)}{\lambda_i} \left\langle\!\!\left\langle \hat{\mathbf{P}}^{(0)}(t) \middle| \frac{\partial \mathcal{L}}{\partial u_i(t)} \middle| \hat{\rho}^{(1)}(t) \right\rangle\!\!\right\rangle$$

gradient optimization

$$\text{minimize} \qquad J_T = 1 - \left\langle\!\left\langle \hat{\rho}(T) \middle| \hat{\mathbf{P}}_{\text{tgt}} \right\rangle\!\right\rangle$$

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optimize using quantum trajectories?

optimal control of trajectories

$$J_T = 1 - \left\langle\!\left\langle \hat{\rho}(T) \middle| \hat{\mathbf{P}}_{\text{tgt}} \right\rangle\!\right\rangle \qquad \longleftarrow \hat{\rho} = \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} \left|\Psi_k\right\rangle \left\langle\Psi_k\right|$$

optimal control of trajectories

$$J_T = 1 - \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} \Big| \underbrace{\langle \Psi_k(T) \mid \Psi_{\text{tgt}} \rangle}_{\equiv \tau_k} \Big|^2$$

optimal control of trajectories

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GRAPE/LBFGS

$$\frac{\partial J_T}{u_{ij}} = - \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} \left[\frac{\partial \tau_k}{\partial u_{ij}} \tau_k^* + \tau_k \left(\frac{\partial \tau_k}{\partial u_{ij}} \right)^* \right]$$

$$\frac{\partial \tau_k}{u_{ij}} = \left\langle \Psi_{\text{tgt}}^{(0)}(t_j) \left| \frac{\partial \hat{\mathbf{U}}_{jk}}{\partial u_{ij}} \right| \Psi_k^{(0)}(t)_{j-1}) \right\rangle$$

optimal control of trajectories

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Krotov's method

Palao, Kosloff, PRA 68, 062308 (2003).

$$\Delta u_i(t) = \frac{S_i(t)}{M\lambda_i} \sum_{k=1}^{M} \underbrace{\mathfrak{Im} \left\langle \chi_k^{(0)}(t) \,\middle|\, \hat{\mathbf{H}}_i \,\middle|\, \Psi_k^{(1)}(t) \right\rangle}_{\equiv \Delta u_{ik}(t)},$$

with $\chi_k^{(0)}(T) = -\frac{\partial J_T}{\partial \langle \Psi_k |} = \tau_k^{(0)} \, |\Psi_{\text{tgt}}\rangle$

optimal control of trajectories

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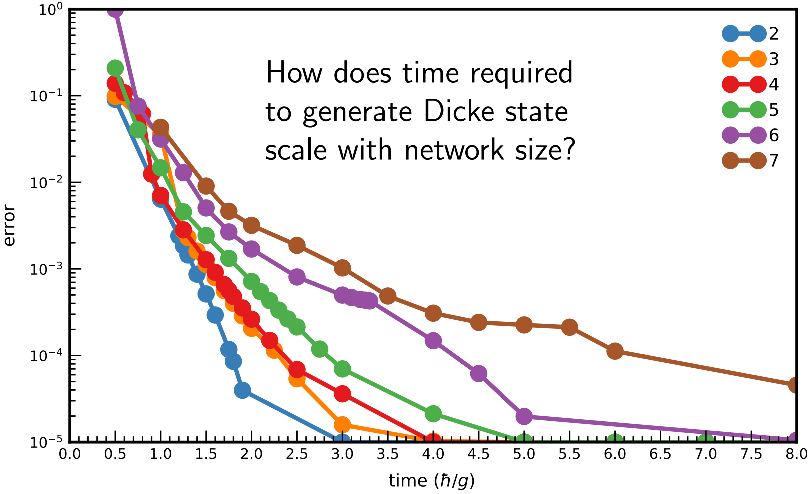
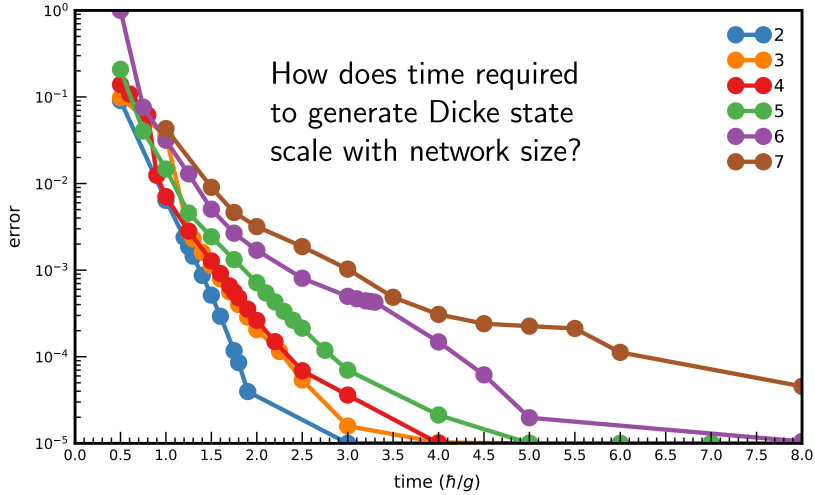
quantum speed limit for Dicke state generation





⇒ single trajectory allows to determine speed limit

caveats

Krotov's method Palao, Kosloff, PRA 68, 062308 (2003).

$$\Delta u_i(t) = \frac{S_i(t)}{M\lambda_i} \sum_{k=1}^{M} \underbrace{\mathfrak{Im} \left\langle \chi_k^{(0)}(t) \,\middle|\, \hat{\mathbf{H}}_i \,\middle|\, \Psi_k^{(1)}(t) \right\rangle}_{\equiv \Delta u_{ik}(t)},$$

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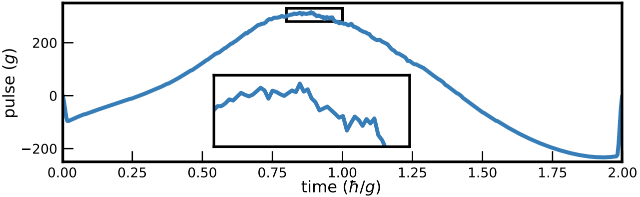
- noisy pulses



$\Rightarrow$ smooth pulses every few iterations

caveats

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Palao, Kosloff, PRA 68, 062308 (2003).

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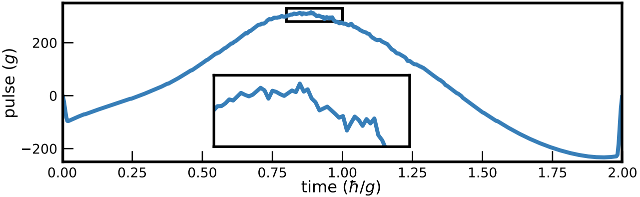


$\Rightarrow$ smooth pulses every few iterations

- no monotonic convergence

Krotov's method (in Liouville space)

$$\Delta u_i(t) = \frac{S_i(t)}{\lambda_i} \left\langle\!\!\left\langle \hat{\mathbf{P}}^{(0)}(t) \middle| \frac{\partial \mathcal{L}}{\partial u_i(t)} \middle| \hat{\boldsymbol{\rho}}^{(1)}(t) \right\rangle\!\!\right\rangle$$

$$\hat{\rho} = \lim_{M \to \infty} \frac{1}{M} \sum_{k=1}^{M} \left|\Psi_k\right\rangle \left\langle\Psi_k\right|$$

Krotov's method (in Liouville space)

$$\Delta u_i(t) = \frac{S_i(t)}{M^2 \lambda_i} \sum_{k,k'=1}^{M} \mathfrak{Im}\Big[\Big\langle \xi_k^{(0)}(t) \Big| \hat{\mathbf{H}}_i \Big| \Psi_{k'}^{(1)}(t) \Big\rangle \times \Big\langle \Psi_{k'}^{(1)}(t) \Big| \xi_k^{(0)}(t) \Big\rangle \Big]$$

$$\text{with } \hat{\mathbf{P}}^{(0)}(t) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} \left|\xi_k\right\rangle \left\langle\xi_k\right|$$

cross-trajectory optimization

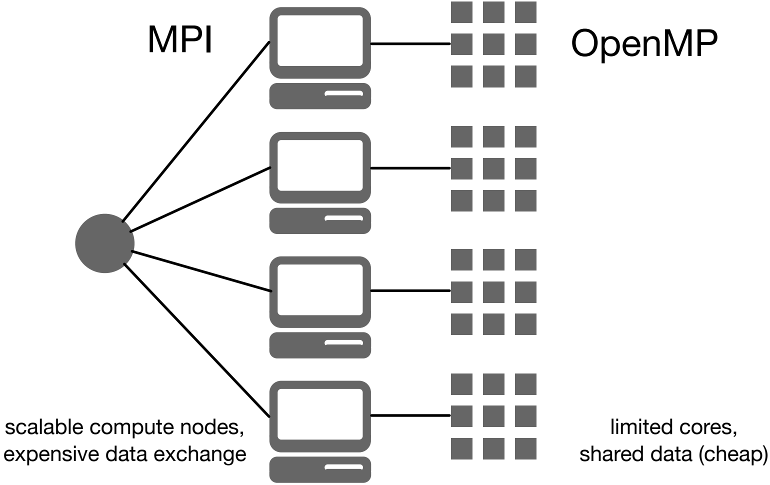
Krotov's method (in Liouville space)

$$
\Delta u_i(t) = \frac{S_i(t)}{M^2 \lambda_i} \sum_{k,k'=1}^{M} \mathfrak{Im}\Big[\Big\langle \xi_k^{(0)}(t) \Big| \hat{\mathbf{H}}_i \Big| \Psi_{k'}^{(1)}(t) \Big\rangle \times \Big\langle \Psi_{k'}^{(1)}(t) \Big| \xi_k^{(0)}(t) \Big\rangle \Big]
$$

$$
\text{with } \hat{\mathbf{P}}^{(0)}(t) = \lim_{M \to \infty} \frac{1}{M} \sum_{k=1}^{M} \left|\xi_k\right\rangle \left\langle \xi_k\right|
$$

cross-trajectory terms!

parallelization approaches



hybrid parallelization of trajectory optimization

Krotov's method

$$\Delta u_i(t) = \frac{S_i(t)}{M^2 \lambda_i} \sum_{k,k'=1}^{M} \mathfrak{Im}\Big[\Big\langle \xi_k^{(0)}(t) \Big| \hat{\mathbf{H}}_i \Big| \Psi_{k'}^{(1)}(t) \Big\rangle \times \Big\langle \Psi_{k'}^{(1)}(t) \Big| \xi_k^{(0)}(t) \Big\rangle \Big]$$

hybrid parallelization of trajectory optimization

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- sending update $\Delta u_i(t)$ is cheap

hybrid parallelization of trajectory optimization

Krotov's method

$$\Delta u_i(t) = \frac{S_i(t)}{M^2 \lambda_i} \sum_{k,k'=1}^{M} \mathfrak{Im}\Big[\Big\langle \xi_k^{(0)}(t) \Big| \hat{\mathbf{H}}_i \Big| \Psi_{k'}^{(1)}(t) \Big\rangle \times \Big\langle \Psi_{k'}^{(1)}(t) \Big| \xi_k^{(0)}(t) \Big\rangle \Big]$$

- sending update $\Delta u_i(t)$ is cheap
- sending states $|\xi_k\rangle$, $|\Psi_{k'}\rangle$ is expensive

hybrid parallelization of trajectory optimization

Krotov's method

$$\Delta u_i(t) = \frac{S_i(t)}{M^2 \lambda_i} \sum_{k,k'=1}^{M} \mathfrak{Im}\Big[\Big\langle \xi_k^{(0)}(t) \Big| \hat{\mathbf{H}}_i \Big| \Psi_{k'}^{(1)}(t) \Big\rangle \times \Big\langle \Psi_{k'}^{(1)}(t) \Big| \xi_k^{(0)}(t) \Big\rangle \Big]$$

- sending update $\Delta u_i(t)$ is cheap
- sending states $|\xi_k\rangle$, $|\Psi_{k'}\rangle$ is expensive

$\Rightarrow$ proposal:
only cross-reference trajectories k, k' that are local

acknowledgments

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conclusion

large Hilbert space & dissipative dynamics
$\rightarrow$ optimal control of MCWF trajectories with Krotov's method

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Kurt Jacobs
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Hideo Mabuchi
Stanford

QNET

https://github.com/mabuchilab/QNET

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Kassel (Germany)

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quantum dynamics and control

https://www.qdyn-library.net