Optimal Controlled Phasegates for Ultracold Atoms in an Optical Lattice at the Quantum Speed Limit

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Summary

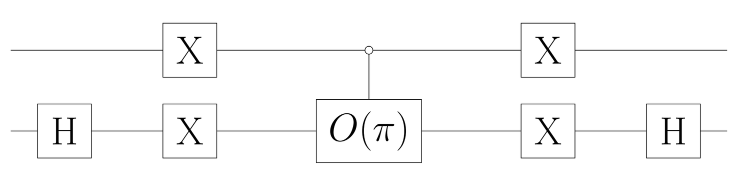
We study controlled phasegates for ultracold atoms in an optical lattice [1]. The qubits are encoded in the electronic states. A shaped laser pulse drives transitions between the ground and electronically excited states where the atoms are subject to a long-range $1/R^3$ interaction. We fully account for this interaction and use optimal control theory to calculate the pulses. This allows us to determine the minimum pulse duration, respectively the gate time T that is required to obtain high fidelity. We find the gate time to be limited either by the interaction strength in the excited state or by the ground state vibrational motion in the trap. The latter needs to be resolved in order to fully restore the motional state of the atoms at the end of the gate.

Universal Quantum Computing

The set of all one-qubit gates plus the two-qubit CNOT is universal. More generally, the CNOT is equivalent to the controlled phasegate, combined with Hadamard and X-gates.

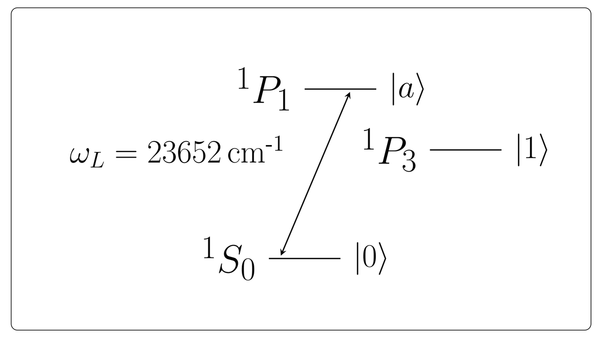
$$\hat{O}(\phi) = \begin{pmatrix} e^{i\phi} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

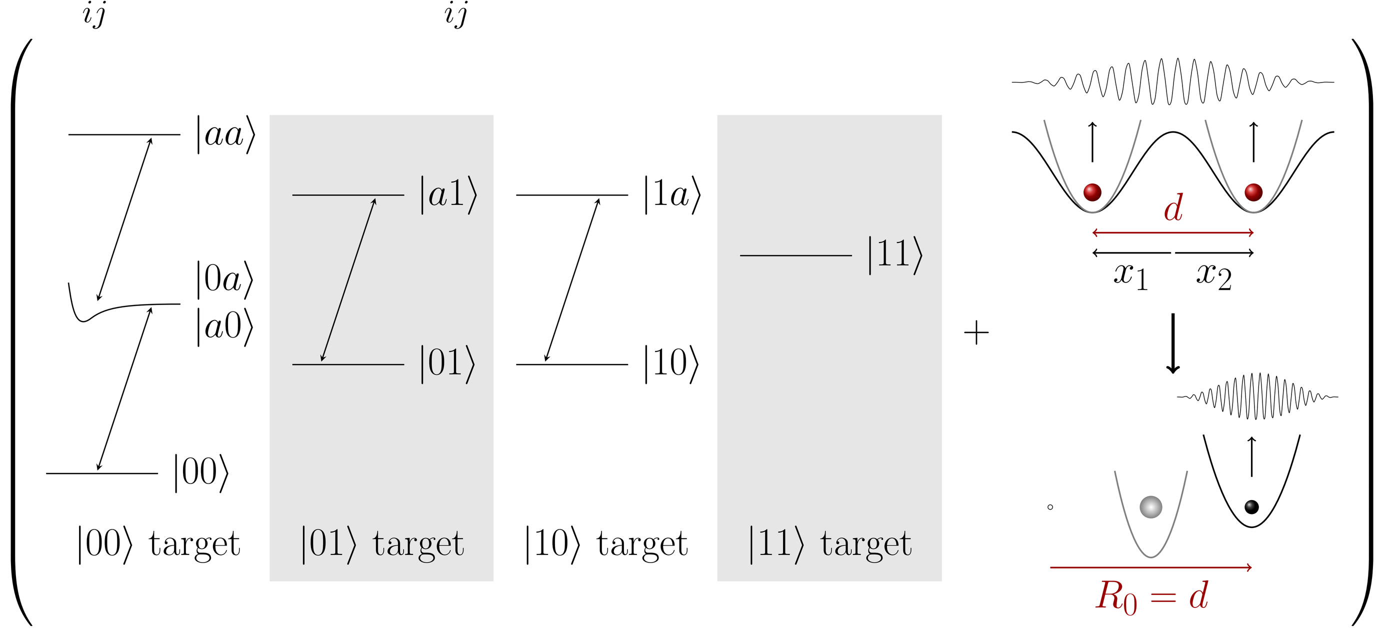
$CNOT =$



Qubit Encoding in Calcium

$$\hat{H}_{2q} = \begin{pmatrix} E_0 & 0 & \mu\epsilon(t) \\ 0 & E_1 & 0 \\ \mu\epsilon(t) & 0 & E_a \end{pmatrix} \otimes \mathbb{1}_{1q} \otimes \mathbb{1}_{x_1} + \mathbb{1}_{1q} \otimes \begin{pmatrix} E_0 & 0 & \mu\epsilon(t) \\ 0 & E_1 & 0 \\ \mu\epsilon(t) & 0 & E_a \end{pmatrix} \otimes \mathbb{1}_{x_2} + \sum_{ij} \hat{V}_{BO}^{(ij)}(|x_2 - x_1|) + \sum_{ij} \hat{V}_{\text{trap}}^{(ij)}(x_1, x_2)$$

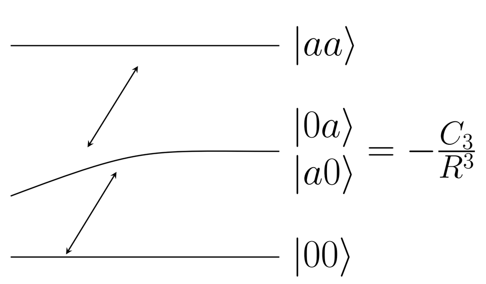




initialize to trap ground state $\Psi_{00,i}(R) \approx \left(\frac{\mu\omega_0}{4\pi\hbar}\right)^{1/4} \sum_{\pm} e^{\frac{-m\omega_0}{2\hbar}(d \pm R)^2}$

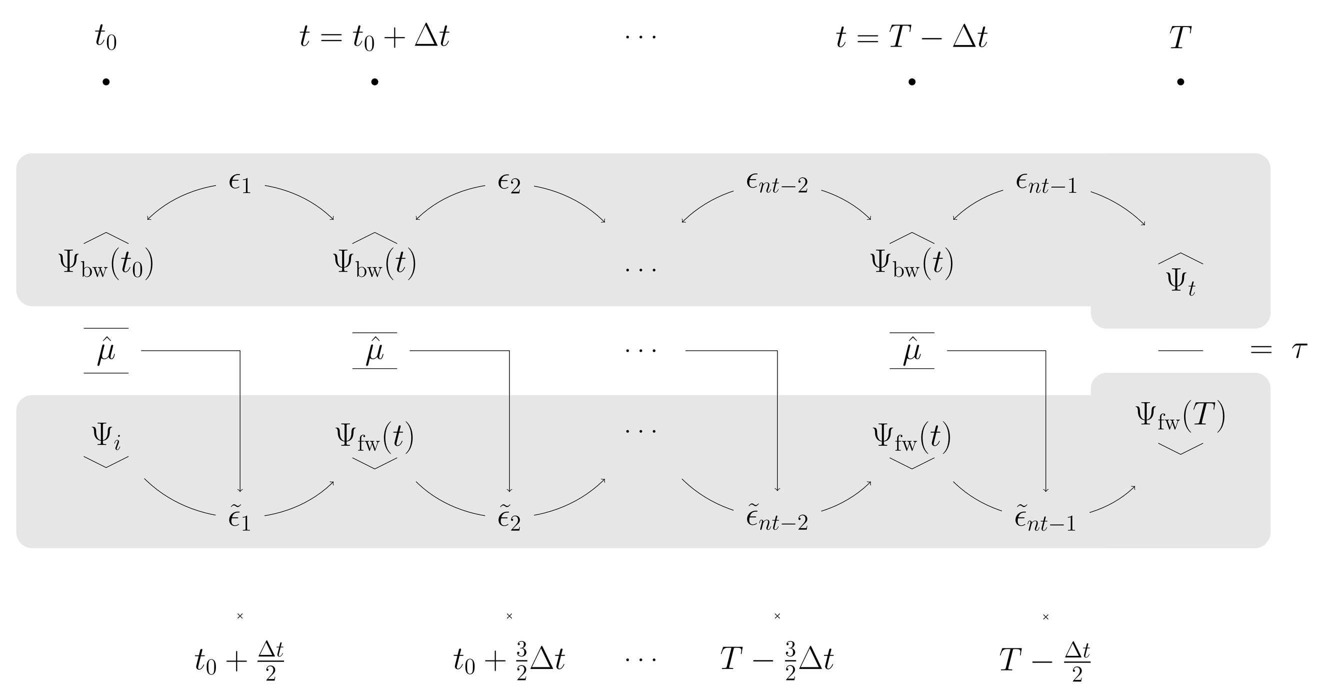
Parameters:

- Use actual calcium $B^1\Sigma_u^+$ interaction potential at $d = 5$ nm.
- Use generic dipole-dipole interaction with variable C_3 at $d = 200$ nm.



OCT: Finding an Optimized Pulse

Change pulse iteratively to minimize [2, 3] $J = -F + \int \frac{\alpha}{S(t)} \Delta\epsilon(t)\,dt$, $F = \frac{1}{N}\Re\left[\text{tr}\left(\hat{O}^\dagger \hat{U}\right)\right]$



$$\Delta\epsilon(\tilde{t}) = \frac{S(\tilde{t})}{\alpha} \Im \left[\sum_{k=1}^{N} \frac{1}{2} \left\langle \Psi_{ik} \left| \hat{O}^\dagger \hat{U}^\dagger (T \to t, \epsilon^{(0)}) \hat{\mu} \hat{U}(0 \to t, \epsilon^{(1)}) \right| \Psi_{ik} \right\rangle \right]$$

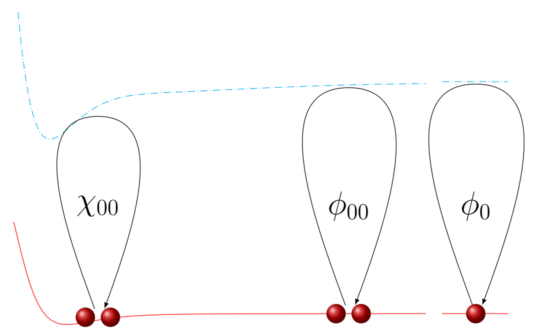
One-Qubit and Two-Qubit Phases

	full optimization scheme	reduced optimization scheme
system phases	$\phi_{00}, \phi_{01}, \phi_{10}, \phi_{11}$	$\phi_{00}, \phi_0, \phi_1$
optimization targets	$\vert 00\rangle \longrightarrow e^{i(\phi+\phi_T)}\vert 00\rangle$ $\vert 01\rangle \longrightarrow e^{i\phi_T}\vert 01\rangle$ $\vert 10\rangle \longrightarrow e^{i\phi_T}\vert 10\rangle$ $\vert 11\rangle \longrightarrow e^{i\phi_T}\vert 11\rangle$	$\vert 00\rangle \longrightarrow e^{i(\phi+\phi_T)}\vert 00\rangle$ $\vert 0\rangle \longrightarrow e^{i\phi_T/2}\vert 0\rangle$ $\vert 1\rangle \longrightarrow e^{i\phi_T/2}\vert 1\rangle$
gate phases	ϕ_{00} $\phi_{10} = \phi_{01}$ ϕ_{11}	$= \phi_{00}$ $= \phi_0 + \phi_1$ $= 2\phi_1$
true two-qubit phase	$\chi = \phi_{00} - \phi_{01} - \phi_{10} + \phi_{11}$	$\chi = \phi_{00} - 2\phi_0$

True two-qubit phase χ from Cartan decomposition [4].

$$\hat{U} = \hat{U}_1 \hat{O}(\chi) \hat{U}_2,$$

where $\hat{U}_1$ and $\hat{U}_2$ are purely local operations.



References

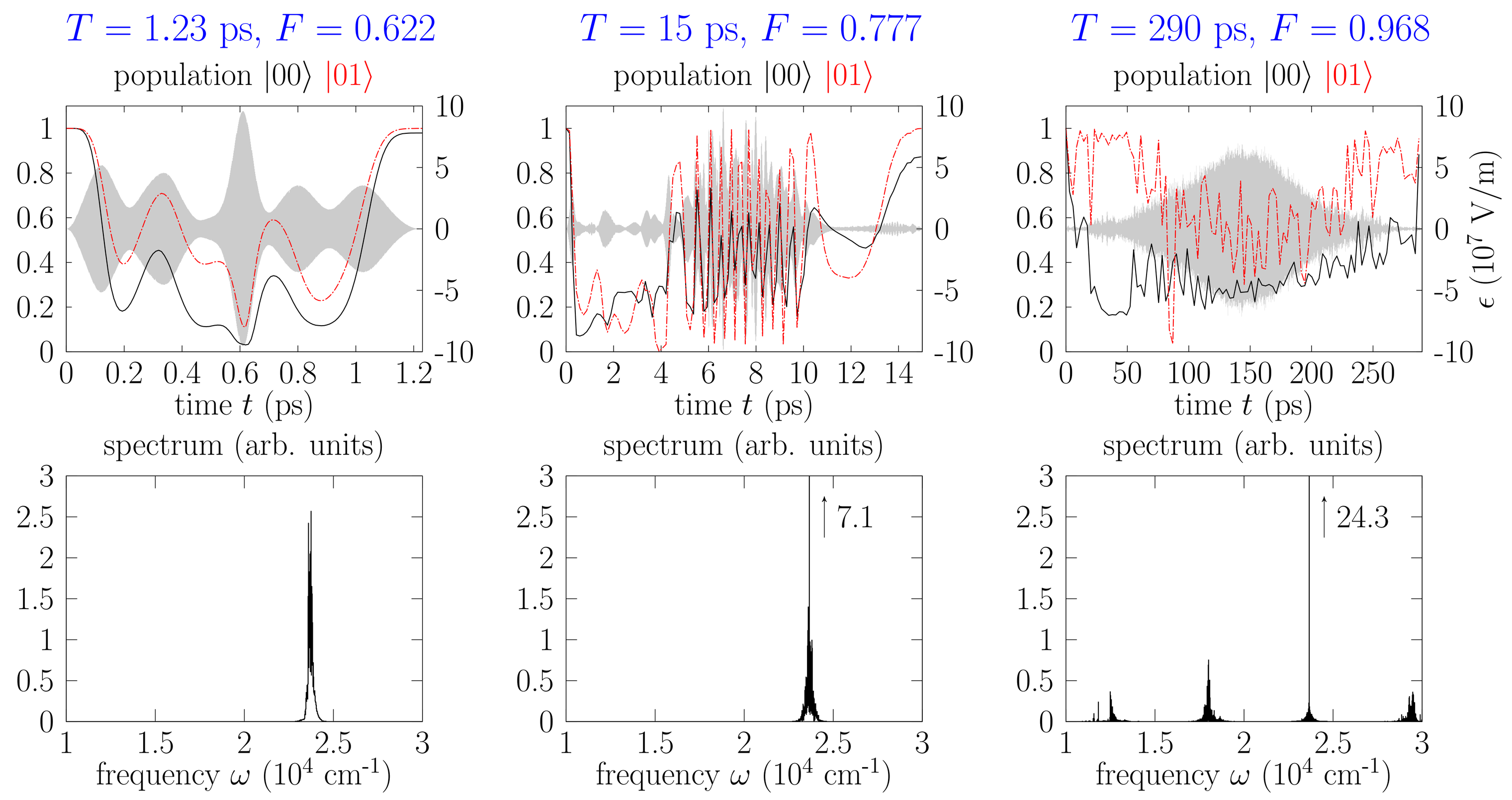
[1] T. Calarco et al., *Phys. Rev. A* **61**, 022304 (2004)

[2] J. P. Palao, R. Kosloff, *Phys. Rev. Lett.* **89**, 188301 (2002), *Phys. Rev. A* **68**, 062308 (2003).

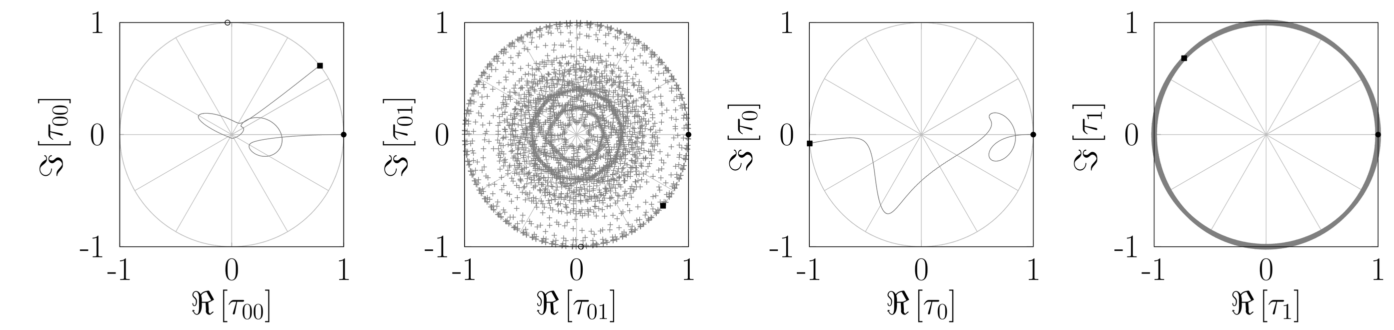
[3] C. P. Koch et. al. *Phys. Rev. A* **70**, 013402 (2004).

[4] J. Zhang et. al. *Phys. Rev. A.* **67**, 042313 (2003).

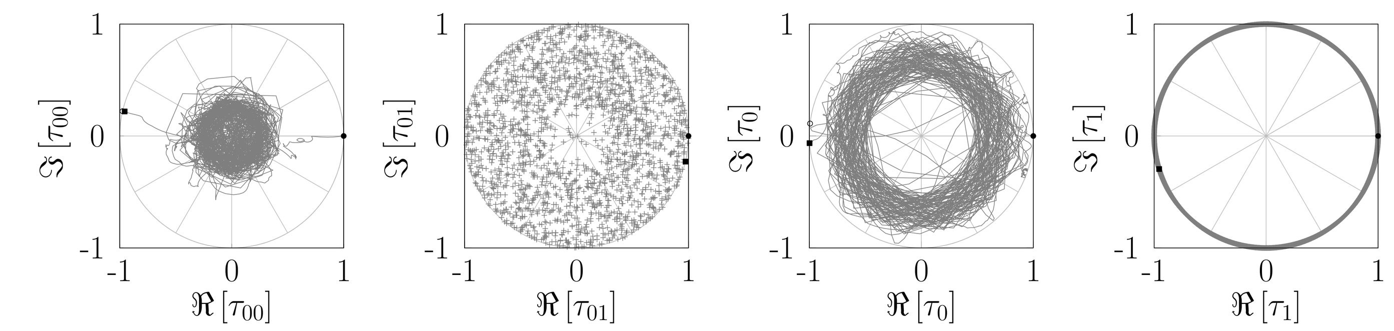
Results



Phase Dynamics for $T = 1.23$ ps in two-qubit and single-qubit system

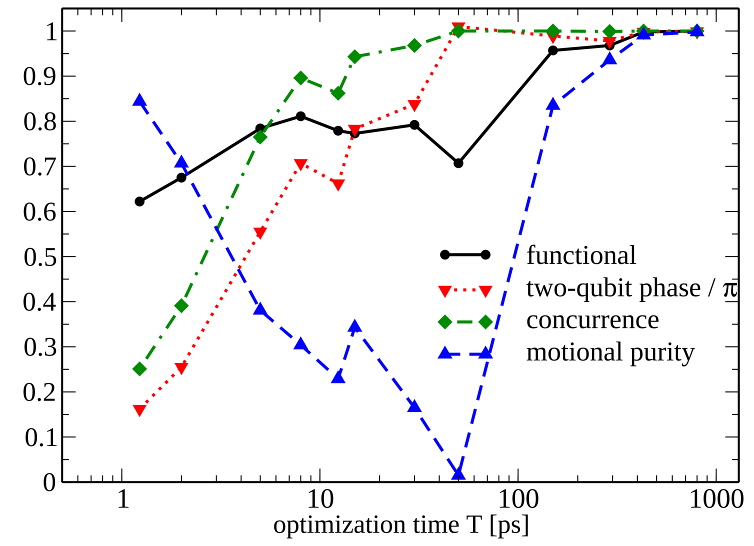


Phase Dynamics for $T = 290$ ps in two-qubit and single-qubit system



$d = 5$ nm (40 iters)

T/ps	F	χ/π	C	F_{00}
1.23	0.622	0.162	0.251	0.844
2.00	0.675	0.255	0.391	0.707
5.00	0.784	0.555	0.765	0.381
8.00	0.811	0.707	0.896	0.304
12.36	0.779	0.660	0.861	0.228
15.00	0.777	0.777	0.940	0.181
30.00	0.792	0.838	0.968	0.165
50.00	0.707	1.010	1.000	0.015
150.00	0.957	0.989	1.000	0.835
290.00	0.968	0.978	0.999	0.936
430.00	0.998	0.998	1.000	0.991
800.00	1.000	0.999	1.000	0.998



$d = 200$ nm (40 iters)

C_3	T/ps	F	χ/π	C	F_{00}
1×10^6	1	0.579	0.023	0.036	0.998
4×10^8	1	0.607	0.810	0.956	0.085
8×10^8	1	0.568	0.866	0.978	0.045
1×10^9	1	0.555	0.874	0.980	0.033
1×10^6	0.5	0.582	0.027	0.042	0.996
4×10^8	0.5	0.684	0.655	0.856	0.289
8×10^8	0.5	0.549	0.853	0.973	0.042
1×10^9	0.5	0.523	0.880	0.982	0.025