Efficient Optimization of Quantum Gates for Rydberg Atoms and Transmon Qubits under Dissipative Evolution

Michael H. Goerz, Daniel M. Reich, Christiane P. Koch

Institut für Physik, Universität Kassel, Kassel, Germany

Quantum Dynamics and Control

UNI KASSEL VERSITÄT

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Abstract

We consider two different physical systems to illustrate an efficient optimization of quantum gates under dissipative evolution, requiring the propagation of only three states, irrespective of the dimension of the Hilbert space. [1] In the first example, two trapped neutral atoms are excited to a Rydberg state, via a decaying intermediary state [2]. The interaction between both atoms in the $|rr\rangle$ state allows for the realization of a diagonal CPHASE gate. Optimal control theory finds a solution that uses a STIRAP-like mechanism to suppress population in the decaying intermediary state, while implementing the desired gate. As a second example, we consider two superconducting transmon qubits [3] coupled via a shared transmission line resonator [4]. The Hamiltonian in this case also allows for non-diagonal gates, and we optimize for a $\sqrt{i\mathrm{SWAP}}$, taking into account energy relaxation and dephasing of the qubits [5]. The system is driven at a frequency close to the center between both qubits, and the optimized gate exploits a near-resonance of the $|0\rangle \to |1\rangle$ transition on the left qubit and the $|1\rangle \to |2\rangle$ transition on the right qubit. For both examples, the gate fidelity reached by optimization is only limited by the decoherence.

① Efficient OCT of a Unitary in Liouville Space

No need to characterize the full dynamical map!

$$\rho^{(1)}_{ij} = \frac{2(N-i+1)}{N(N+1)}\delta_{ij} \Rightarrow \frac{1}{20}\begin{pmatrix} 8&0&0&0\\ 0&6&0&0\\ 0&0&4&0\\ 0&0&0&2 \end{pmatrix}, \quad \rho^{(2)}_{ij} = \frac{1}{N} \Rightarrow \frac{1}{4}\begin{pmatrix} 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1\\ 1&1&1&1 \end{pmatrix}, \quad \rho^{(3)}_{ij} = \frac{1}{N}\delta_{ij} \Rightarrow \frac{1}{4}\begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix}$$

"populations" "phases" "subspace"

$\hat{\boldsymbol{\rho}}_1$: Are we diagonal in the correct basis?
$\hat{\boldsymbol{\rho}}_2$: totally rotated state → relative phases } check & distinguish unitaries
$\hat{\boldsymbol{\rho}}_3$: Do we have a (unital) dynamical map on the logical subspace?

Optimization Functional:

$$J = J_T - \lambda_a \int_0^T \frac{[\epsilon(t) - \epsilon_{\text{ref}}(t)]^2}{S(t)}\,\mathrm{d}t; \qquad J_T = 1 - \sum_{j=1}^{3} \frac{w_j}{\mathrm{Tr}[\rho_j^2(0)]} \mathrm{Tr}\left[\hat{\mathbf{O}}\hat{\rho}_j\hat{\mathbf{O}}^\dagger \mathcal{D}[\hat{\rho}_j]\right]$$

- J_T becomes 0 if (and only if) $\mathcal{D}$ implements target gate $\hat{\mathbf{O}}$
- different states can have different weights w_i

Control equations: for Krotov method [7], with $\epsilon_{\text{ref}} = \epsilon_{\text{old}}$

$$\frac{d\hat{\boldsymbol{\rho}}_i}{dt} = -i[\hat{\mathbf{H}}, \hat{\boldsymbol{\rho}}_i] + \mathcal{L}_D(\hat{\boldsymbol{\rho}}_i) \tag{1}$$

$$\frac{d\hat{\boldsymbol{\sigma}}_i}{dt} = -i[\hat{\mathbf{H}}, \hat{\boldsymbol{\sigma}}_i] - \mathcal{L}_D(\hat{\boldsymbol{\sigma}}_i) \quad \text{and} \quad \hat{\boldsymbol{\sigma}}_i(t=T) = \frac{w_i}{\mathrm{Tr}[\hat{\boldsymbol{\rho}}_i^2(0)]}\hat{\mathbf{O}}\hat{\boldsymbol{\rho}}_i(0)\hat{\mathbf{O}}^\dagger, \tag{2}$$

$$\Delta\epsilon(t) = \frac{S(t)}{\lambda_a}\sum_{i=1}^{n} \mathfrak{Im}\left\{\mathrm{Tr}\left(\hat{\boldsymbol{\sigma}}_i^{\text{old}}(t)\left.\frac{\partial\mathcal{L}(\hat{\boldsymbol{\rho}}_i)}{\partial\epsilon}\right|_{\rho_i^{\text{new}},\epsilon^{\text{new}}}\right)\right\} \quad \text{with} \quad \frac{\partial\mathcal{L}(\hat{\boldsymbol{\rho}})}{\partial\epsilon} = -i\left[\frac{\partial\hat{\mathbf{H}}}{\partial\epsilon}, \hat{\boldsymbol{\rho}}\right] \tag{3}$$

Dissipation in examples is modeled as master equation in Lindblad form, with

$$\mathcal{L}_D(\hat{\boldsymbol{\rho}}) = \sum_j \gamma_j D[\hat{\mathbf{A}}_j]\hat{\boldsymbol{\rho}}; \qquad D[\hat{\mathbf{A}}]\hat{\boldsymbol{\rho}} = \hat{\mathbf{A}}\hat{\boldsymbol{\rho}}\hat{\mathbf{A}}^\dagger - \frac{1}{2}\left(\hat{\mathbf{A}}^\dagger\hat{\mathbf{A}}\hat{\boldsymbol{\rho}} + \hat{\boldsymbol{\rho}}\hat{\mathbf{A}}^\dagger\hat{\mathbf{A}}\right)$$

Note: method does *not* depend on equation of motion or model for dissipation!

Measure of merit: average gate fidelity

$$F_{\text{avg}} = \int \langle\Psi|\hat{\mathbf{O}}^\dagger\mathcal{D}\left(|\Psi\rangle\langle\Psi|\right)\hat{\mathbf{O}}|\Psi\rangle\,\mathrm{d}\Psi$$
$$= \frac{1}{d(d+1)}\sum_{i,j=1}^{d}\left(\langle\varphi_i|\hat{\mathbf{O}}^\dagger\mathcal{D}\left(|\varphi_i\rangle\langle\varphi_j|\right)\hat{\mathbf{O}}|\varphi_j\rangle + \mathrm{Tr}\left[\hat{\mathbf{O}}\,|\varphi_i\rangle\langle\varphi_i|\,\hat{\mathbf{O}}^\dagger\mathcal{D}\left(|\varphi_j\rangle\langle\varphi_j|\right)\right]\right)$$

$\hat{\boldsymbol{\rho}}_1$ and $\hat{\boldsymbol{\rho}}_3$ are mixed states ⇒ possibly faster convergence by using set of pure states

- $d+1$ states: expand $\hat{\boldsymbol{\rho}}_1$, keep $\hat{\boldsymbol{\rho}}_2$, expansion of $\hat{\boldsymbol{\rho}}_1$ makes $\hat{\boldsymbol{\rho}}_3$ obsolete
  For two-qubit gate: $\hat{\boldsymbol{\rho}}_1 \to \{|00\rangle\langle 00|, |01\rangle\langle 01|, |10\rangle\langle 10|, |11\rangle\langle 11|\}$
- $2d$ states: expand $\hat{\boldsymbol{\rho}}_1$, plus pure states for mutually unbiased basis (MUB)
  For two-qubit gate:
  - $\hat{\boldsymbol{\rho}}_1 \to \{|00\rangle\langle 00|, |01\rangle\langle 01|, |10\rangle\langle 10|, |11\rangle\langle 11|\}$
  - MUB: $|\tilde{\varphi}_1\rangle = (|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$ $\quad|\tilde{\varphi}_2\rangle = (|00\rangle - |01\rangle + |10\rangle - |11\rangle)/2$
    $|\tilde{\varphi}_3\rangle = (|00\rangle + |01\rangle - |10\rangle - |11\rangle)/2$ $\quad|\tilde{\varphi}_4\rangle = (|00\rangle - |01\rangle - |10\rangle + |11\rangle)/2$

References

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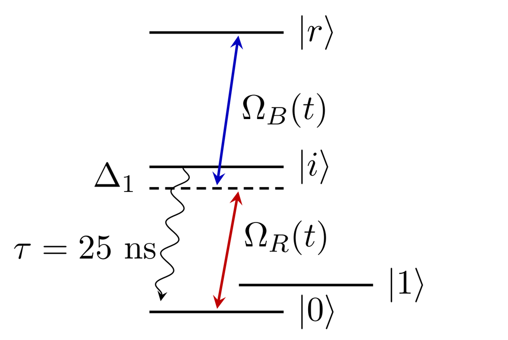
② Optimization of a Rydberg Gate (CPHASE)



Only allows for diagonal gates!

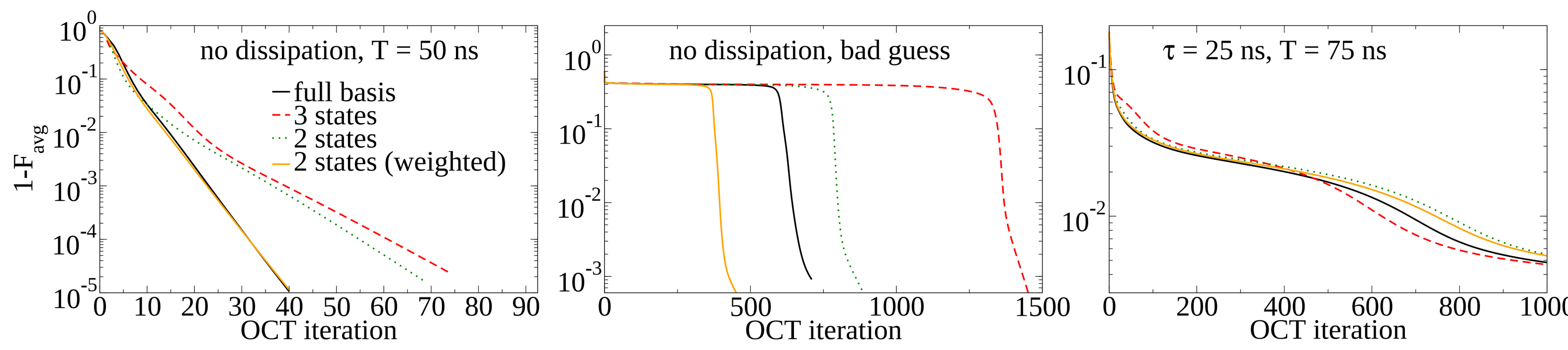
In the RWA:

$$\hat{\mathbf{H}}_{1\text{q}} = \begin{pmatrix} 0 & 0 & \frac{1}{2}\Omega_R(t) & 0 \\ 0 & E1 & 0 & 0 \\ \frac{1}{2}\Omega_R(t) & 0 & \Delta_1 & \frac{1}{2}\Omega_B(t) \\ 0 & 0 & \frac{1}{2}\Omega_B(t) & 0 \end{pmatrix}$$

Two-qubit Hamiltonian:

$$\hat{\mathbf{H}}_{2q} = \hat{\mathbf{H}}_{1q} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbf{H}}_{1q} - U\,|rr\rangle\langle rr|$$

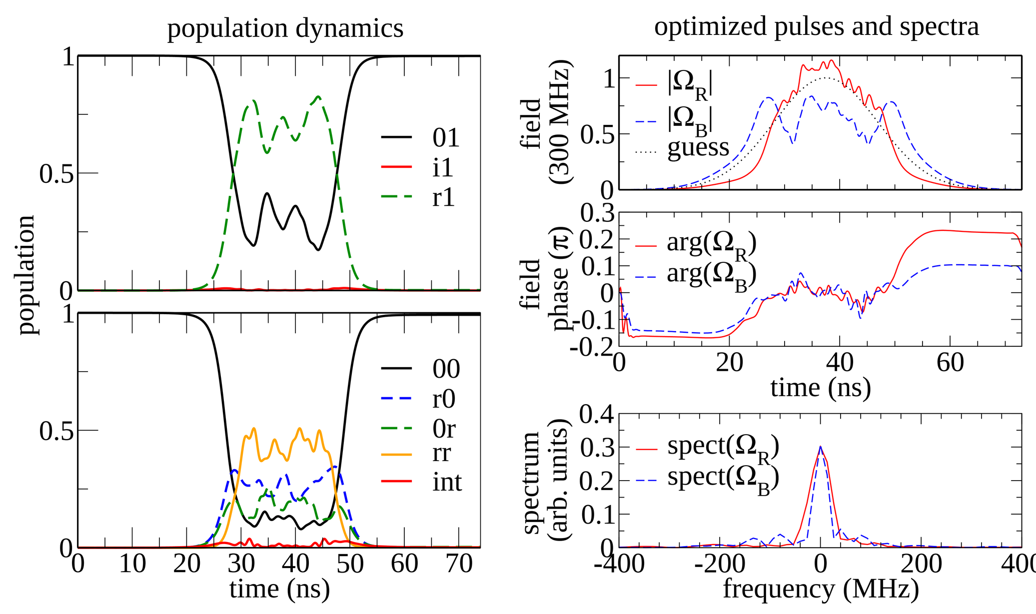
Optimization Results



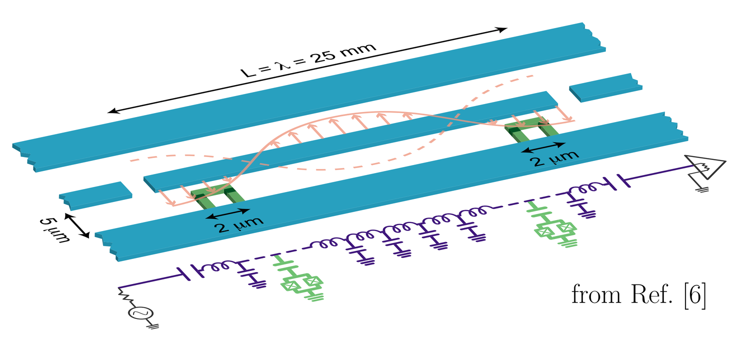
Parameters:

Δ_1	600 MHz
Δ_2	0
E_1	6.8 GHz
Ω_R, Ω_B	300 MHz
U	50 MHz
$\tau = 1/\gamma$	25 ns

- STIRAP-like mechanism → suppress int. pop.
- center oscillations: relative phases



③ Optimization of a Transmon Gate ($\sqrt{i\mathrm{SWAP}}$)



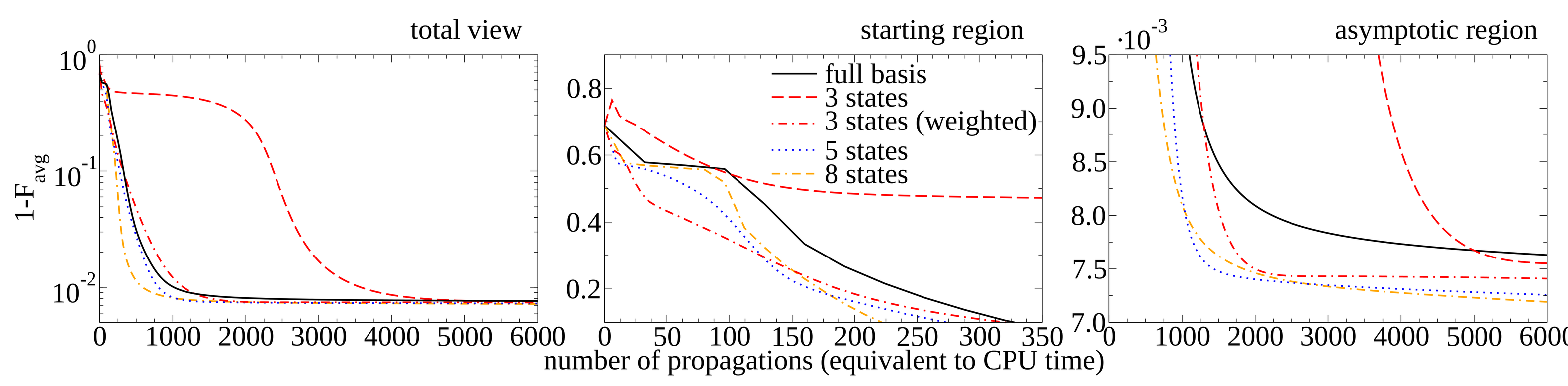
qubit frequency ω_1	4.3796 GHz
qubit frequency ω_2	4.6137 GHz
drive frequency ω_d	4.4985 GHz
anharmonicity δ_1	-239.3 MHz
anharmonicity δ_2	-242.8 MHz
effective qubit-qubit coupling J	-2.3 MHz
qubit 1 decay time T_1	38.0 μs
qubit 2 decay time T_1	32.0 μs
qubit 1 dephasing time T_2^*	29.5 μs
qubit 2 dephasing time T_2^*	16.0 μs

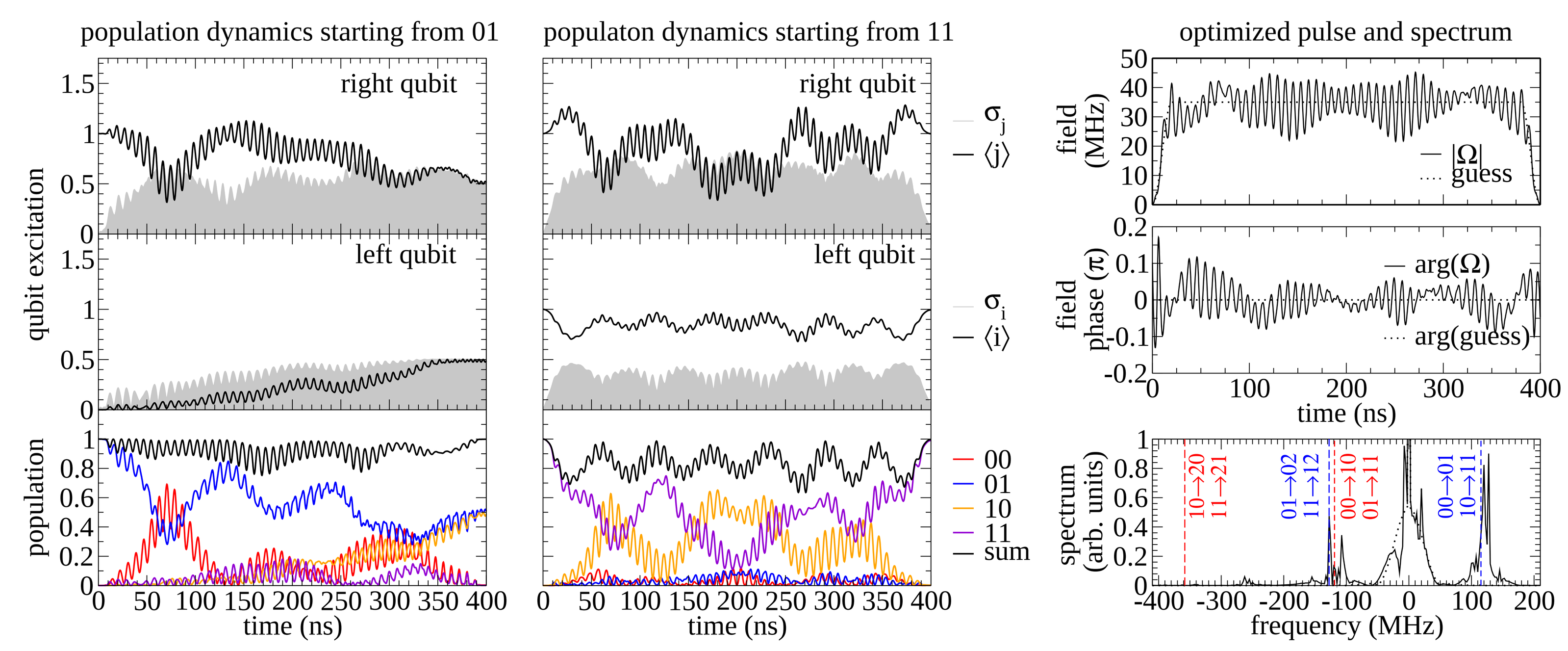
Cavity mediates static interaction between the qubits, driven excitation of each qubit

Effective Hamiltonian (cavity integrated out):

$$\hat{\mathbf{H}} = \left(\omega_1 - \frac{\delta_1}{2}\right)\hat{\mathbf{b}}_1^\dagger\hat{\mathbf{b}}_1 + \frac{\delta_1}{2}\left(\hat{\mathbf{b}}_1^\dagger\hat{\mathbf{b}}_1\right)^2 + \left(\omega_2 - \frac{\delta_2}{2}\right)\hat{\mathbf{b}}_2^\dagger\hat{\mathbf{b}}_2 + \frac{\delta_2}{2}\left(\hat{\mathbf{b}}_2^\dagger\hat{\mathbf{b}}_2\right)^2 + J\left(\hat{\mathbf{b}}_1^\dagger\hat{\mathbf{b}}_2 + \hat{\mathbf{b}}_1\hat{\mathbf{b}}_2^\dagger\right)$$
$$+\,\Omega(t)\cos(\omega_d t)\left(\hat{\mathbf{b}}_1 + \hat{\mathbf{b}}_1^\dagger + \hat{\mathbf{b}}_2 + \hat{\mathbf{b}}_2^\dagger\right)$$

Optimization Results





④ Conclusions

- A set of three states is sufficient for gate optimization, independent of dimension of Hilbert space
- Further reduction possible for restricted dynamics, e.g. Hamiltonians only allowing diagonal gates
- Choosing proper weights for the optimization states improves convergence
- For two-qubit gates, savings in both CPU time and memory by a factor of 8; even more savings for larger Hilbert spaces

⇒ Gate optimization in open quantum systems with large Hilbert spaces have become significantly more feasible