Implementation of a Controlled Phasegate for Ultracold Calcium Atoms Using Optimal Control

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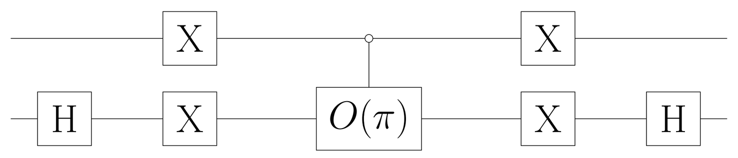
Summary

We consider a quantum computational system of ultracold Calcium atoms in an optical lattice [1]. The qubits are encoded in the electronic states. A shaped laser pulse couples $|0\rangle$ to an auxilliary state $|a\rangle$. For the two-qubit system of two neighboring atoms, there is a dipole-type $(1/R^3)$ interacting for the $|0a\rangle$ state, allowing the implementation of a controlled phasegate on the $|00\rangle$ state. Via optimal control, for different pulse lengths T, we attempt to find laser pulses that yield the target transformation. At very short trap distances (5 nm), we find that for short pulses, a two-qubit phase can be reached; however, the atoms couple to the motional degree of freedoms. Only for very long pulses that can resolve the trap frequency, good fidelities can be found.

Universal Quantum Computing

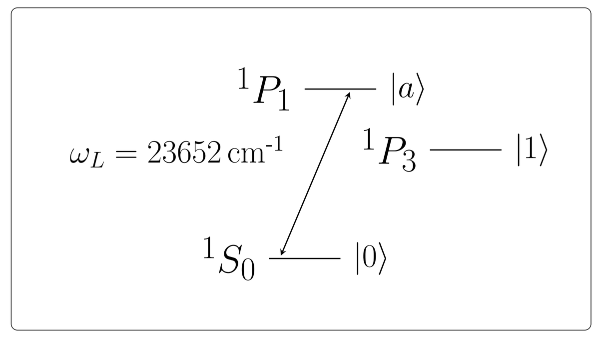
The set of all one-qubit gates plus the two-qubit CNOT is universal. More generally, the CNOT is equivalent to the controlled phasegate, combined with Hadamard and X-gates.

$$\hat{O}(\phi) = \begin{pmatrix} e^{i\phi} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \qquad CNOT =$$



Qubit Encoding in Calcium

$$\hat{H}_{2q} = \begin{pmatrix} E_0 & 0 & \mu\epsilon(t) \\ 0 & E_1 & 0 \\ \mu\epsilon(t) & 0 & E_a \end{pmatrix} \otimes \mathbb{1}_{1q} \otimes \mathbb{1}_{x_1} + + \mathbb{1}_{x_2} \otimes \mathbb{1}_{1q} \otimes \begin{pmatrix} E_0 & 0 & \mu\epsilon(t) \\ 0 & E_1 & 0 \\ \mu\epsilon(t) & 0 & E_a \end{pmatrix} + + \sum_{ij} \hat{V}^{(ij)}_{BO}(|x_2 - x_1|) + \sum_{ij} \hat{V}^{(ij)}_{\text{trap}}(x_1, x_2)$$



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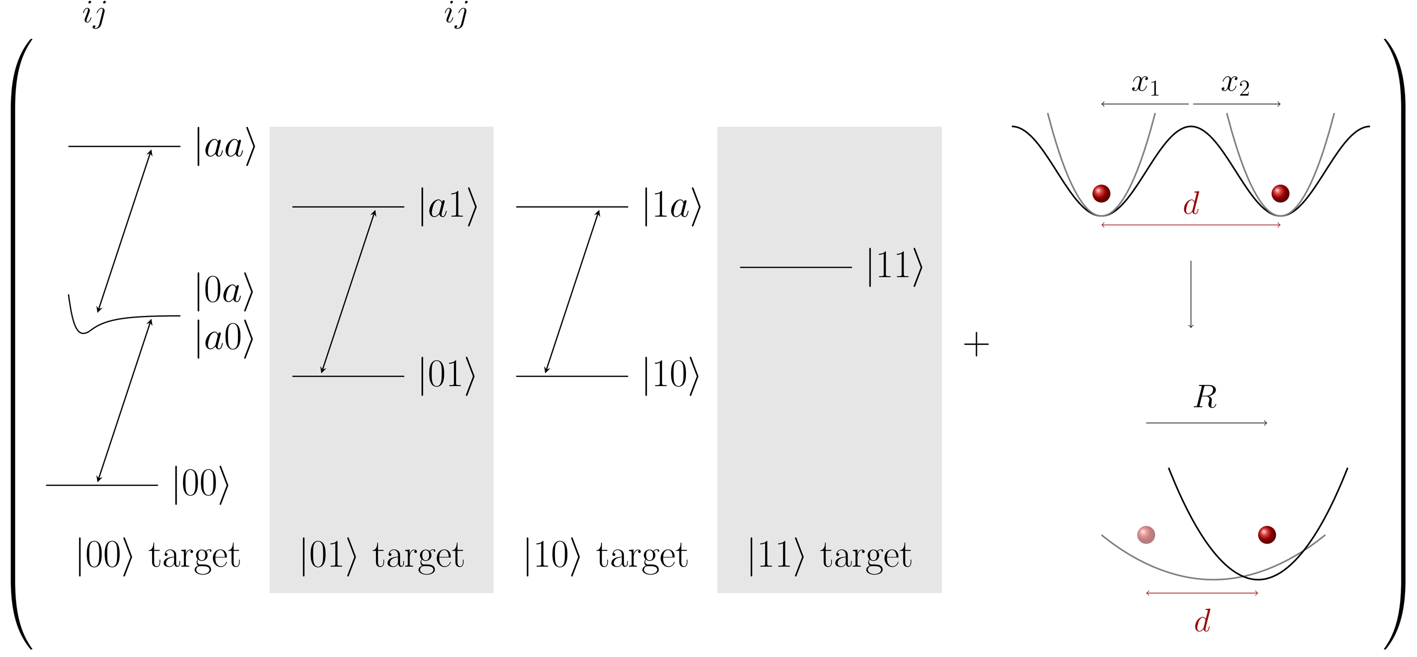


initialize to trap ground state $\quad \Psi_{00,i}(R) \approx \left(\frac{\mu\omega_0}{4\pi\hbar}\right)^{1/4} \sum_{\pm} e^{\frac{-m\omega_0}{2\hbar}(d \pm R)^2}$

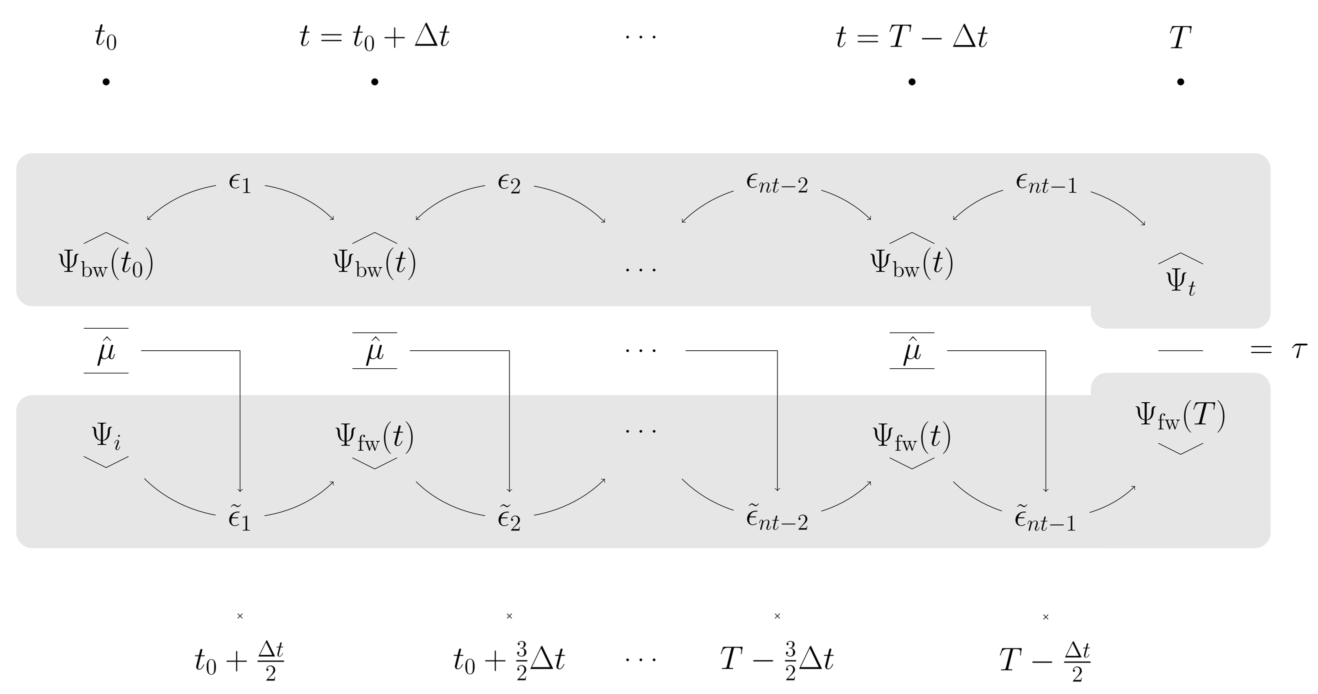
Parameters:

- Spatial grid: $R_{\min} = 5.0$ a.u., $R_{\max} = 300.0$ a.u., $nr = 512$
- Trap parameters: $\omega = 400$ MHz, $d = 5$ nm.
- Optimize for different gate times: medium $T = 1.23 - 50$ ps, long $T = 150 - 800$ ps, and short $T = 0.1 - 1.0$ ps

Look at different targets, and use one-photon or three-photon guess pulses; Use Gaussian guess pulses that perform full Rabi cycles in the given time window.

OCT: Finding an Optimized Pulse

Change pulse iteratively to minimize[2, 3] $\quad J = -F + \int \frac{\alpha}{S(t)} \Delta\epsilon(t)\,\mathrm{d}t, \; F = \frac{1}{N} \Re\left[\mathrm{tr}\left(\hat{O}^\dagger \hat{U}\right)\right]$



$$\Delta\epsilon(\tilde{t}) = \frac{S(\tilde{t})}{\alpha} \Im \left[\sum_{k=1}^{N} \frac{1}{2} \left\langle \Psi_{ik} \middle| \hat{O}^\dagger \hat{U}^\dagger (T \to t, \epsilon^{(0)}) \hat{\mu} \hat{U}(0 \to t, \epsilon^{(1)}) \middle| \Psi_{ik} \right\rangle \right]$$

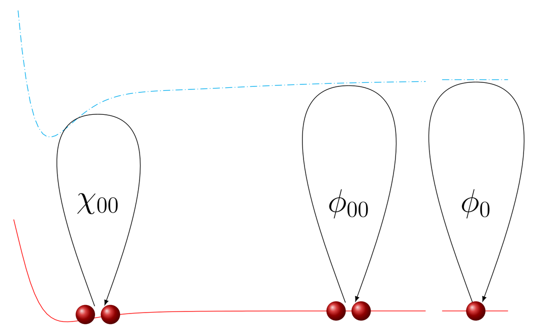
One-Qubit and Two-Qubit Phases

	full optimization scheme	reduced optimization scheme
system phases	ϕ_{00}, ϕ_{01}, ϕ_{10}, (ϕ_{11})	ϕ_{00}, ϕ_0, (ϕ_1)
optimization targets	$\|00\rangle \longrightarrow e^{i(\phi+\phi_T)}\|00\rangle$ $\|01\rangle \longrightarrow e^{i\phi_T}\|01\rangle$ $\|10\rangle \longrightarrow e^{i\phi_T}\|10\rangle$	$\|00\rangle \longrightarrow e^{i(\phi+\phi_T)}\|00\rangle$ $\|0\rangle \longrightarrow e^{i\phi_T/2}\|0\rangle$
gate phases	ϕ_{00} $\phi_{10} = \phi_{01}$ ϕ_{11}	$= \phi_{00}$ $= \phi_0 + \phi_1$ $= 2\phi_1$
true two-qubit phase	$\chi = \phi_{00} - \phi_{01} - \phi_{10} + \phi_{11}$	$\chi = \phi_{00} - 2\phi_0$

True two-qubit phase χ from Cartan decomposition [4].

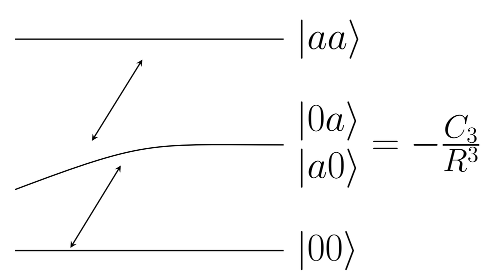
$$\hat{U} = \hat{U}_1 \hat{O}(\chi) \hat{U}_2,$$

where $\hat{U}_1$ and $\hat{U}_2$ are purely local operations.



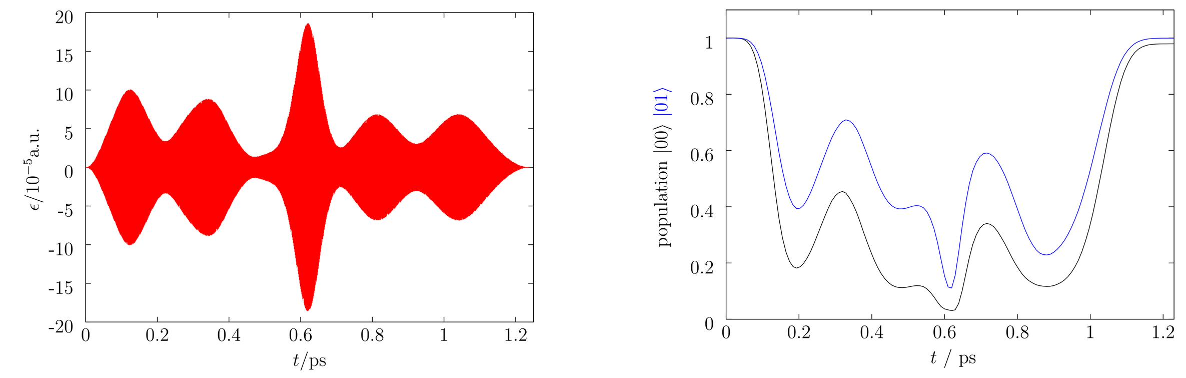
Dipole Interaction

Is it possible to get a two-qubit phasegate in arbitrarily short time if we make the dipole interaction sufficiently strong?
At $d = 200$ nm, try:
$C_3 = 1 \times 10^6, 4 \times 10^8, 8 \times 10^8, 1.6 \times 10^9$.

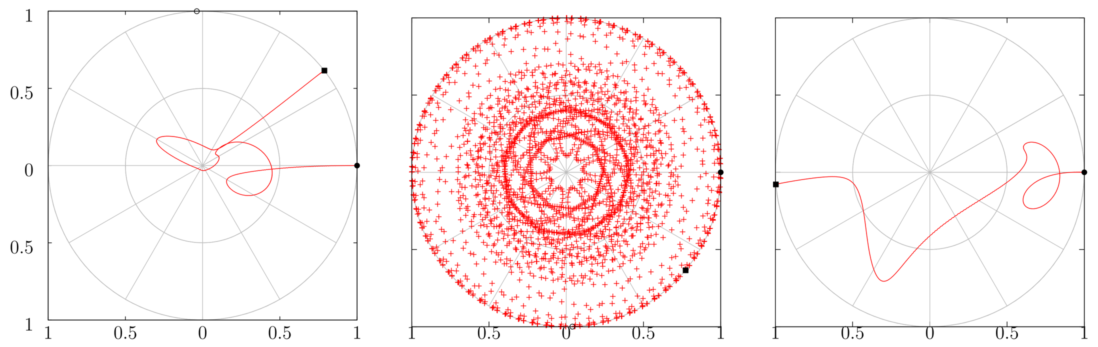


Results

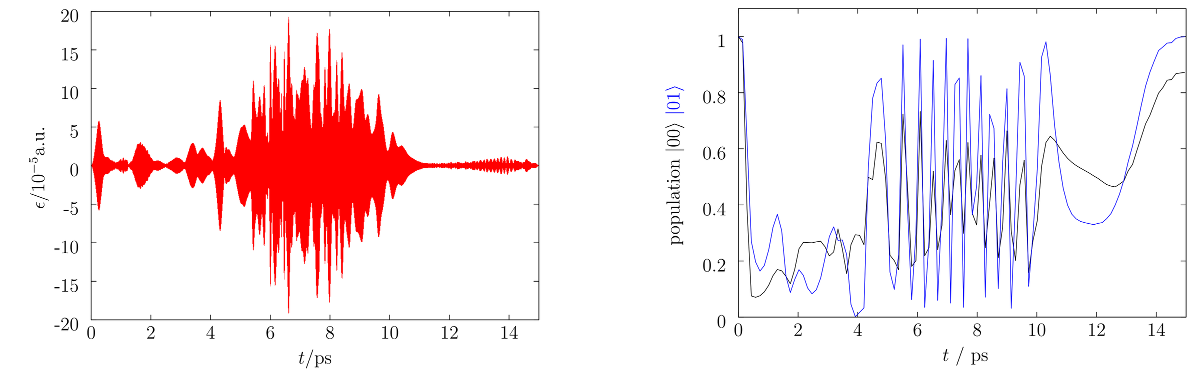
$T = 1.23$ ps
$F = 0.622$
41 iterations



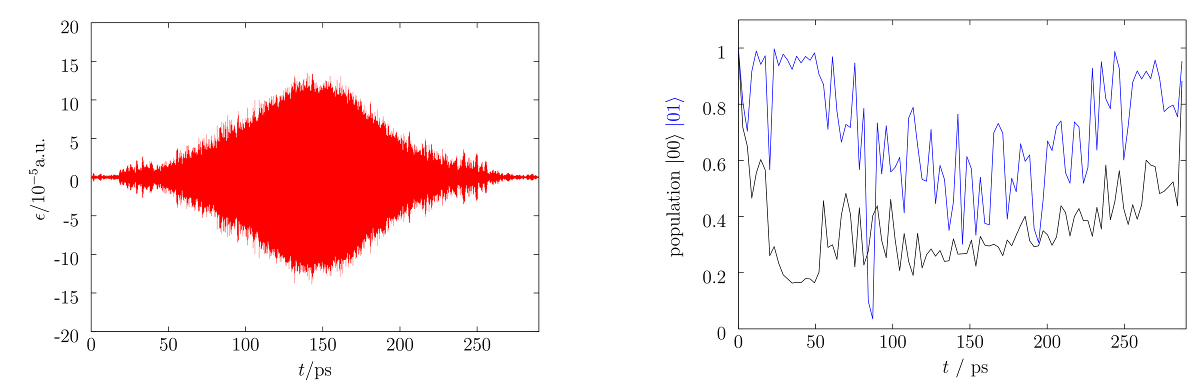
$T = 1.23$ ps
phase dynamics
$(|00\rangle, |01\rangle, |0\rangle)$



$T = 15$ ps
$F = 0.773$
200 iterations



$T = 290$ ps
$F = 0.984$
70 iterations



T/ps	iters	F	χ/π	$\|00\rangle$ pur.
1.23	41	0.622	0.162	0.844
2.00	15	0.639	0.190	0.807
5.00	15	0.719	0.354	0.589
8.00	15	0.787	0.560	0.367
12.36	50	0.779	0.662	0.229
15.00	200	0.773	0.783	0.343
30.00	4	0.630	0.174	0.014
50.00	10	0.653	0.266	0.000
150.00	20	0.898	0.982	0.639
290.00	70	0.984	1.004	0.936
430.00	40	0.998	0.998	0.991
800.00	30	0.999	0.998	0.997

T/ps	C_3	iters	F	χ/π	$\|00\rangle$ pur.
0.5	1×10^6	40	0.582	0.027	0.996
0.5	4×10^8	40	0.684	0.655	0.289
0.5	8×10^8	40	0.549	0.853	0.042
0.5	1.6×10^9	40	0.523	0.880	0.025
1.0	1×10^6	40	0.579	0.023	0.998
1.0	4×10^8	40	0.607	0.810	0.085
1.0	8×10^8	40	0.568	0.866	0.045
1.0	1.6×10^9	40	0.555	0.874	0.033

All optimizations for $\phi = \pi$.

References

[1] T. Calarco et al., *Phys. Rev. A* **61**, 022304 (2004)

[2] J. P. Palao, R. Kosloff, *Phys. Rev. Lett.* **89**, 188301 (2002), *Phys. Rev. A* **68**, 062308 (2003).

[3] C. P. Koch et. al. *Phys. Rev. A* **70**, 013402 (2004).

[4] J. Zhang et. al. *Phys. Rev. A.* **67**, 042313 (2003).